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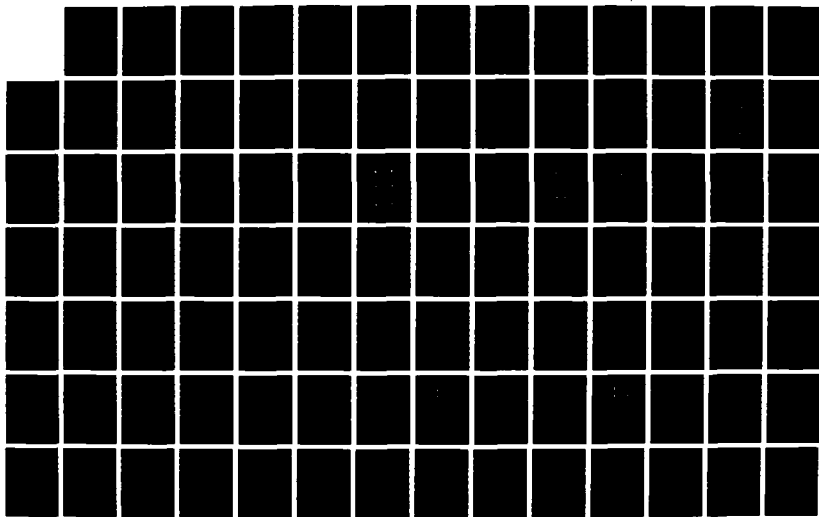
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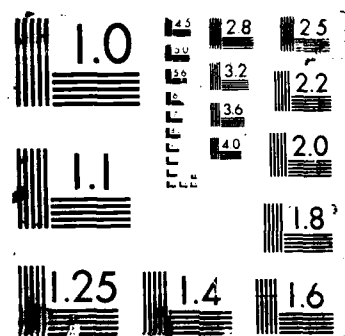
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A METHODOLOGY FOR DETERMINING
THE OPTIMUM MIX OF
ESCORT AND STRIKE AIRCRAFT
IN A TACTICAL STRIKE FORMATION

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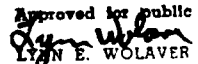
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ESCORT AND STRIKE AIRCRAFT
IN A TACTICAL STRIKE FORMATION

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Operations Research

Muhammad Avais
Squadron Leader, PAF

March 1988

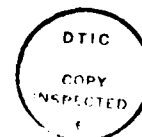
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Muhammad Avais



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ABSTRACT

The purpose of this research was to develop a methodology to aid a decision maker in determining the optimum mix of escort and strike aircraft in a tactical strike formation. The research had four main objectives: (1) Analyze likely air combat between enemy fighter interceptors and offensive aircraft formation. (2) Develop a mathematical model of likely air combat, which could determine the expected outcomes as a function of decision variable--the force mix. (3) Develop a computer program for the model. (4) Provide guidelines for decision making regarding the force mix.

An analysis of likely air combat between the interceptors and the offensive formation was conducted. The basic assumption was that in an interception against the offensive formation, the first event would be a BVR (Beyond Visual Range) missile attack by the interceptors, followed by a close-range combat. The analysis concluded that the close-range air combat could be broken down further into a series of relatively simple events, where each event had a certain probability of occurrence. The events of the air combat formed a hierarchy. The number of aircraft destroyed and the number of aircraft available with the formation were defined as the "outcomes" of a combat. The probabilities of the outcomes were determined from the event probabilities by folding back the event hierarchy.

To account for the effect of more than one interception,

the number of aircraft in the offensive formation was modelled as a "finite state Markov Chain", with transitions occurring as a result of combat engagements. The transition probabilities for the "Markov Chain" were derived from the probabilities of the outcomes of individual engagements. The number of friendly aircraft destroyed in engagement were calculated as "expected rewards" associated with the transitions; and the expected number of aircraft available with the offensive formation were calculated from the transition probability distribution.

A computer program was developed for the model, which can generate the required output data for a given force mix.

For decision making regarding the force mix, the Analytic Hierarchy Process (AHP), a methodology based on the Multiple Criteria Decision Theory, was considered the most suitable.

A METHODOLOGY FOR DETERMINING THE OPTIMUM MIX OF ESCORT AND STRIKE AIRCRAFT IN A TACTICAL STRIKE FORMATION

I. INTRODUCTION

BACKGROUND

Tactical air strikes are an important aspect of joint air-land warfare. The purpose of tactical air strikes is to destroy the targets behind the forward edge of battle area which have a direct influence on the land battle. Since access to such targets requires a penetration through the enemy air defense system, the success of a mission depends upon the survivability of the attacking force against the air defense threats.

The presence of escort aircraft with a strike formation can greatly increase the formation's ability to survive the most significant threat in an air defense system--the fighter interceptors. As an example of how the presence of escort aircraft could affect the survivability of the strike force, consider the following situation. Two alternatives are available. Plan I is to send the strike formation without any escort cover. The aircraft are configured for ground attack and have a limited air to air capability. Plan II involves sending the same number of aircraft, but not all of them are configured for ground attack, some are configured for air to air combat.

In Plan I, if the formation encounters the interceptors,

the probability of losing a certain number of aircraft would be considerably higher than Plan II due to the inability of the aircraft to out-fight the interceptors. Whereas in Plan II, if interceptors are encountered, the aircraft configured for air to air combat could effectively engage the interceptors and reduce the loss of aircraft. Even though the expected bomb load delivered on the target would be smaller in Plan II, the number of aircraft returning safely from a mission would be increased. The proportion of the strike formation dedicated to defense would determine the formation's ability to survive against the interceptors, and the ultimate outcome of the mission. This example illustrates the importance of the decision regarding the force mix of escort and strike aircraft.

The aircraft operated by a tactical air force are usually of the type which can be configured for either role. For example, aircraft like the F-4, F-5, F-15 and F-16 are equally capable of performing a ground attack or an air-to-air mission. In the event of a war, a day to day decision must be made regarding the mission configurations, to ensure an efficient employment of aircraft.

There are several factors that need to be considered in this respect, for example, the effectiveness of the escorts, the ability of the attack aircraft to self-defend, the reliability of any threat warning system carried on-board, and the kill probability of the enemy interceptors. This complex problem can be greatly simplified by estimating the implications of a

particular decision.

The focal point of this research is to develop a methodology which can aid a decision maker attempting to resolve this issue. A decision maker, in this case, would like to know the expected losses and the expected number of attack aircraft reaching the target for a particular force mix. The most suitable aid would be a formula or a computer program which could calculate the expected outcomes by using some known parameters.

PURPOSE OF RESEARCH

The purpose of this research is to devise a methodology which can simplify the process of decision making regarding the appropriate mix of attack and escort aircraft, in a tactical attack formation.

SPECIFIC OBJECTIVE

The specific objective of this research is to model the combat between the interceptors and the aircraft of the penetrating formation, and generate the required output data for decision making.

SUBSIDIARY OBJECTIVES

In order to achieve the main objective, it would be necessary to meet the following sub-objectives:

- (a) Analyze the process of air combat that is likely to take place between the interceptors and the attack aircraft.
- (b) Construct a mathematical model of the combat between the interceptors and the attack aircraft, which could

generate the following output data:

- (i) The expected number of friendly aircraft destroyed during the engagements.
- (ii) The expected number of aircraft surviving with the attack formation after the engagements.
- (c) Develop the computer code for that model.
- (d) Provide guidelines for decision making.

METHODOLOGY

An important sub-objective of this research is to model the air combat between the offensive aircraft and the interceptors so that inferences could be drawn regarding the losses and the survivability of strike and escort aircraft. An analytical model, in this case, is considered preferable to a computer simulation model because of its inherent accuracy and simplicity. The model should:

- (a) Calculate the expected losses of aircraft and the expected number of aircraft able to continue with the mission after a given number of interceptions.
- (b) Account for the kill probability of the interceptor.
- (c) Incorporate the simultaneous interaction of more than two aircraft in a combat.
- (d) Account for the ability of the attack aircraft to defend against the interceptors.
- (e) Base the output on known or predictable factors. For example, the reliability of the detection devices carried on board etc.

The existing analytic as well as simulation models do not meet these requirements; therefore, a new model is developed. (The reasons for such unsuitability are discussed in Chapter II.)

A detailed analysis of the likely air combat scenarios is conducted. The discussion concludes that an air combat between the interceptors and the penetrators can be modeled as a series of events. The outcomes of the events are either probabilistic in nature or are determined by human decisions. The events thus form a "hierarchy". The probabilities of the individual events are assumed to be known to a decision maker apriori. The losses sustained by the friendly aircraft are defined as the final outcomes of a combat engagement. The unconditional probabilities of the final outcomes are calculated by folding back the event hierarchy.

To account for the effect of more than one interception, the process is modelled as a Markov Chain. Each interception is regarded as a transition, and the state space for the system is defined by the number of escort and the striker aircraft in the offensive formation. The "one step" transition probabilities for the Markov Process are computed from the event hierarchy. The "nth" transition matrix (the transition matrix after "n" interceptions) provides the probability distribution for the outcomes after the "n" interception. The aircraft losses are computed as the "rewards" associated with each transition and the losses are accumulated for all the transitions.

A computer program is developed for the model, which

generates the necessary output. For decision making, the Analytical Hierarchy Process (AHP) is employed to help decision maker in selecting the best option.

FORMAT

Chapter II of this paper gives a review of some of the existing models which deal with bomber/interceptor interaction.

Chapter III describes an analysis of the likely air combat between the penetrators and the interceptors and the necessary assumptions made in that context.

Chapter IV presents the theoretical basis for the model. The main subjects under discussion are: (1) the effect of BVR encounters, and (2) the effect of close range engagements.

Chapter V describes the salient features of the computer program developed for the model. An example is also added to demonstrate the use of the model.

Chapter VI outlines a methodology for decision making.

Chapter VII provides a recap of the paper.

II. REVIEW OF THE EXISTING MODELS

Over recent years a great deal has been written about bomber penetration modeling. It has been promoted as a subject to study the bomber survivability against various air defense threats. This chapter gives a brief review of the currently existing models, both analytic and simulation, which deal with this subject. While the computer models provide flexibility to simulate complex air defense scenarios, the analytic models have the advantage of providing a closed form solution to the problem of bomber survivability. The models discussed in this text are designed for specific roles, therefore, they do not have a general applicability. Each model has an area of emphasis.

ANALYTIC MODELS

ARSENAL EXCHANGE MODEL: This model was developed by the Martin-Marietta corporation and is used at Headquarters USAF, Studies and Analysis (AFCSA). It is an expected value model to study the structure of total strategic forces. It calculates the probability of a successful bomber penetration P_{PM} as:

$$P_{PM} = (1 - P_E) + P_E(1 - P_{AI})^{1/I}$$

where

P_E - probability of encountering the parameter defense

P_{AI} - probability that a bomber is killed by single
interceptor pass

I - total number of interceptors

B - total number of bombers

Several different techniques are used to optimize weapon allocation and the model can treat full force allocation, a variety of defenses and force design problems. A number of scenarios ranging from a single strike against military targets to three strike games involving problems of selecting a weapons reserve or a value target reserve for the initiators third strike may be analyzed using the model (1:12-13).

The model is not considered suitable for this research because it does not meet the basic requirements of this research (as specified in the previous chapter). For example, the model does not incorporate simultaneous interaction of more than two aircraft in a combat and the ability of a penetrating aircraft to take any evasive action against the interceptor.

CODE-50. This is a widely used aggregated model developed by the Lambda Corporation. It is capable of handling a mixture of offensive weapons types. In the model, the bomber penetration probability, which describes a bomber's chance of surviving against a specific type of fighter, is assumed to be proportional to

$$\exp[-a(F/B)^c]$$

where F is the number of fighters, B is the number of bombers, a and c are model inputs that incorporate the effect of most of the parameters affecting bomber penetration. The model is over simplified in that it is difficult to find appropriate values for a and c that will adequately represent the parameters. Due to the same reason the model is not suitable for this research (1:13).

COLLIDE: COLLIDE is an aggregate conversion model for air combat, designed to assess the impact of command and control on fighter bomber engagements. The output of the model is a probability for target detection and interceptor conversion under different engagement scenarios.

This model by itself cannot be used for this research because of its own specific purpose. However, its output, can be used to calculate the expected number of interceptions against a given raid which in turn can be used as an input to the intended model for this research (1:14).

COPEM-1 (Corridor penetration model): COPEM-1 was developed at Stanford Research Institute as part of a study to improve the representation of airborne strategic systems in aggregated effectiveness evaluation models. It is a time dependent engagement model. Its purpose is to generate average bomber penetration probabilities as a function of the depth of penetration along a single corridor into a defended area. The underlying assumption in the model is that the number of intercepts that occur on a bomber follow a Poisson distribution with a time dependent parameter. This parameter is calculated iteratively at discrete time intervals during the engagement (1:14).

The analytic solution for bomber survivability against the manned interceptors and its suitability for this research will be discussed later.

THE SCHULTIS MODEL: The Schultis model, titled as "A National-Level Analytic Model for Penetration of Various Combined Air Defense Deployments by Cruise Missiles or Bombers," is a small expected value penetration model. Five types of air defenses are modelled: forward air defense, barrier SAMs, random area SAMs, fighter interceptors, and terminal SAMs. The basic approach taken in the model is to separate the defenses into bands that are penetrated sequentially by the bombers. It deals with large numbers of penetrators and relies on saturation of the defenses, rather than leakage, as the primary method of penetration. It is based on the assumption that the offensive penetrator's best strategy is to attack in files along narrow corridors instead of individually at random. As the files approach each line of defense, losses may be expected at first, however, the SAMs will exhaust their missiles or the fighters within range of a particular file will exhaust their AAMs and in effect a path through any particular layer of defenses will be cleared for the remaining penetrators. The overall effectiveness of the model is measured by the total value extracted by a given number of penetrators that have survived the bands of defenses (5:12).

Glenn P. Clemens, in his theses, has further developed the Schultis Model to incorporate the variance calculation. He has also employed the basic concepts of the COPEM-1 model to derive the probability of bomber survival against the manned interceptors.

He states that:

$$P_r(X) = [P_{kr}L/P_{kr} \cdot (L-A)] \cdot \text{EXP}(-A \cdot X/V) - [A/P_{kr} \cdot (L-A)] \cdot \text{EXP}(P_{kr} \cdot L \cdot X/V)$$

Where $P_r(X)$ - the probability that a bomber survives through a distance X

P_{kr} - the probability that an interceptor detects, converts and kills the bomber

L - the number of interceptors expected enroute

A - a measure of the capability of the radar net to detect the bomber

V - the speed of the bomber

(1:70-81).

This analytic function has a great advantage of simplicity, but it has a limited application. Because it models one-on-one confrontations, it cannot account for an interaction of more than two aircraft engaged in a combat. Therefore, any of the models employing this particular solution, would not meet the criteria for this research.

HISTVEC: This is a fast running expected value model of bomber penetration. Fighters and bombers are both modelled in detail. In particular, the model considers fighter air base deployment, different fighter types with different detection and conversion probabilities that are functions of both fighter type and penetration altitude. Decoy considerations such as flight range, credibility, threat dilution and primary payload displacement are all incorporated in the model. The model has limited application due to its inability to account for the

possible detection of the interceptors and evasive maneuvers executed by the penetrator aircraft (3:16-19).

LULEJIAN-MARKOVIAN: This model, developed by Lulejian and Associates, uses Markov process to model the bomber penetration problem. Both fighters and decoys are modelled in as great detail as in HISTVEC with the added feature that fighter can be reassigned while airborne. SAM systems are modelled with two different SAM types allowed. Firing rate limits and degradation by chaff or ECM are also included in the model (3:15-16). The model has the same limitation as for the HISTVEC.

SIMULATION MODELS

ADVANCED PENETRATION MODEL (APM): APM was originally developed by Boeing Corporation for Headquarters USAF, the Assistant Chief of Staff, Studies and Analyses. The model is capable of simulating a strategic mission of the entire bomber and tanker force from take off to recovery. It consists of two main parts, a Mission Planner and an Air Battle Simulator. The overall mission scenario is user defined; the model then generates individual flight plans for each bomber or tanker in the force. Various rules or constraints may be imposed. The plan for each sortie includes routing, refueling, target allocation and recovery (2:85-88).

The model is not suitable for small scale simulations because a vast amount of data is required to be input, maintenance of computer routines is time consuming, and a single run may take more than twenty hours of computer time (2:88).

SPEED (Simulation of Penetrator Encountering Extensive Defence): SPEED was developed by the Calspan Corporation and is a large Monte Carlo simulation of bomber penetration through air defense systems. It is a fast running model and simulates fighters, SAMs, ECM, and ground controlled intercepts, and economically generates histories of events. It also summarizes engagements and the outcomes, which allows examination of the results of engagements between individual offensive and defensive weapons (6:2-4).

In order to calculate the losses of the bomber force the model takes into consideration only the PK (probability of kill) of single interceptor against single bomber. It does not incorporate multiple aircraft interaction. The model, therefore, is not suitable for this research.

OTHER MODELS

In addition to the above, a few more existing models are listed here:

- (a) **Beta Cadens** --a very large simulation model that includes all strategic forces and detailed damage assessment information (3:13).
- (b) **NYLAND (RAND)** -- an expected value model that includes bombers, decoys, interceptors and SAMs (3:13-16).
- (c) **PEGASOS** -- an expected value model which includes, interceptors, SAM defenses, bomber decoys, and ICBMs (3:13-16).

III. A TACTICAL STRIKE MISSION

INTRODUCTION

The discussion in this chapter is focussed on the conduct of a typical strike mission and the events of an interception. The analysis identifies the expected course of action for both the hostile and the friendly sides in this context. The concepts and the assumptions made will then be applied to model the combat between the interceptors and the offensive aircraft.

AN OVERVIEW OF A TACTICAL STRIKE MISSION

The strike formation. A strike mission is normally planned with four to eight aircraft per formation. Some of the aircraft are configured for air to air combat to act as the escorts, and the rest are for ground attack. The aircraft fly in elements and each element normally consists of two aircraft. Elements are semi-independent in a sense that they can operate independently if required but would stay with the parent formation under normal circumstances.

The formation layout is an important aspect of the mission planning because it determines the vulnerability of the formation to the interceptors. While planning the layout, the endeavor is to position the escorts where the strikers get maximum protection, because the escorts have a better maneuverability and combat effectiveness as compared to the strike aircraft. A typical formation layouts is shown in Figure 3.1.

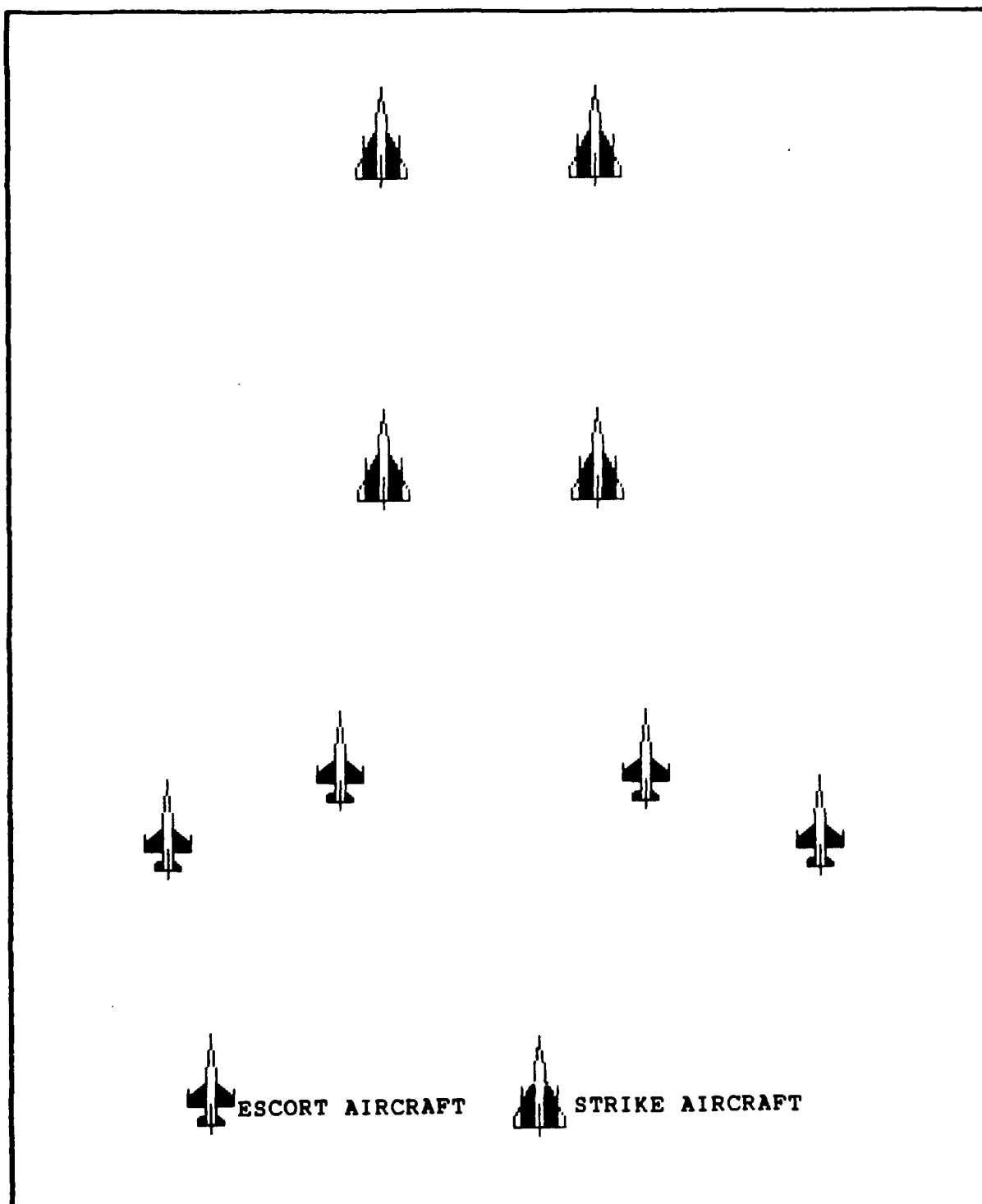


FIGURE 3.1 A TYPICAL FORMATION LAYOUT (DISTANCES ARE NOT TO THE SCALE)

The aircraft in the formation may be equipped with a Radar system or a RHAW (Radar Homing And Warning) system, for detecting a threat.

The Ingress Phase. The ingress phase involves a penetration through the enemy air defense system, where the enemy interceptors are likely to be encountered. The success of the entire mission depends on the strike formation's ability to survive in the enemy air defense area. The offensive formation employs tactics to avoid detection by the enemy radar. However, an engagement with the enemy interceptors is always possible. The formation members keep a lookout for the airborne interceptors. An early detection of the interceptors may also be provided by the onboard Radar and the RHAW system.

If the interceptors make a successful interception, they first launch the BVR (Beyond visual range) missiles and subsequently close in for a short range attack. The offensive formation's strategy is to let the escort aircraft engage the interceptors and let the strike aircraft continue with their mission. However, under certain circumstances the strike aircraft may also be forced to engage. In an engagement, some aircraft may be shot down from both sides. The surviving attack-aircraft cannot rejoin with their parent formation and, therefore, have to return to a home base. The same process is repeated for any number of interceptions, and the strike aircraft which manage to avoid engagements reach the target.

The Attack Phase. After reaching the target, the strike aircraft deliver the bomb load according to their plan. The target area is generally defended by the surface to air missiles and the anti aircraft guns. The interceptors stay outside the terminal defence area to let the ground defenses fire unrestricted.

The Egress Phase. The egress phase is similar to the ingress phase except that after the attack the strike aircraft are cleaned up, therefore, they exit at much faster speed. The increased speed limit gives them an added advantage against the interceptors.

Tied and Free Escorts. As stated earlier, the purpose of the escorts is to protect the strikers from the interceptors. In certain situations it may be advantageous to fly the escorts on an independent route where they can engage the interceptors with an advantage. The escorts in this case are considered "free" The tied escorts are the ones which stay with the formation throughout the conduct of the mission.

THE ENEMY REACTION

The interceptor aircraft are operated under the control and surveillance of an air defense radar system. Since the single aircraft are highly vulnerable in combat, the interceptors are flown as elements of two aircraft. Multiple pairs are deployed against a raid of a large size; and each interceptor pair is independently directed to an offensive formation of aircraft.

The interceptors may be equipped with one or more of the

following:

- (a) BVR (Beyond Visual Range) missiles.
- (b) Short range all aspect missiles.
- (c) Short range rear aspect missiles or guns.

BVR missiles, if installed, are launched before any close combat initiates. For the "rear aspect" and the "all aspect" short range missiles, the interceptors have to close in to a visual range. If the target aircraft take an evasive action then a combat initiates. The outcomes of the combat depend on the effectiveness of the participating aircraft.

THE EVENTS OF AN INTERCEPTION

An interception by nature is a fluid interactive situation. However for analysis, typical interception can be broken down into a series of relatively simple events. Each event has mutually exclusive outcomes which are either probabilistic in nature or are determined by human decisions. The events thus form a "hierarchy," which is shown in Figures 3.2 to 3.5. The process of interception can be well understood by considering each event individually.

BVR Missile Attack. The first event after the initiation of an interception is the BVR missiles attack. The outcomes of the BVR engagement (the number of aircraft destroyed) depend on the number of missiles launched and the probability of kill of a BVR missile.

Close range engagement. The BVR missile attack is followed by a close range engagement. After closing in to a

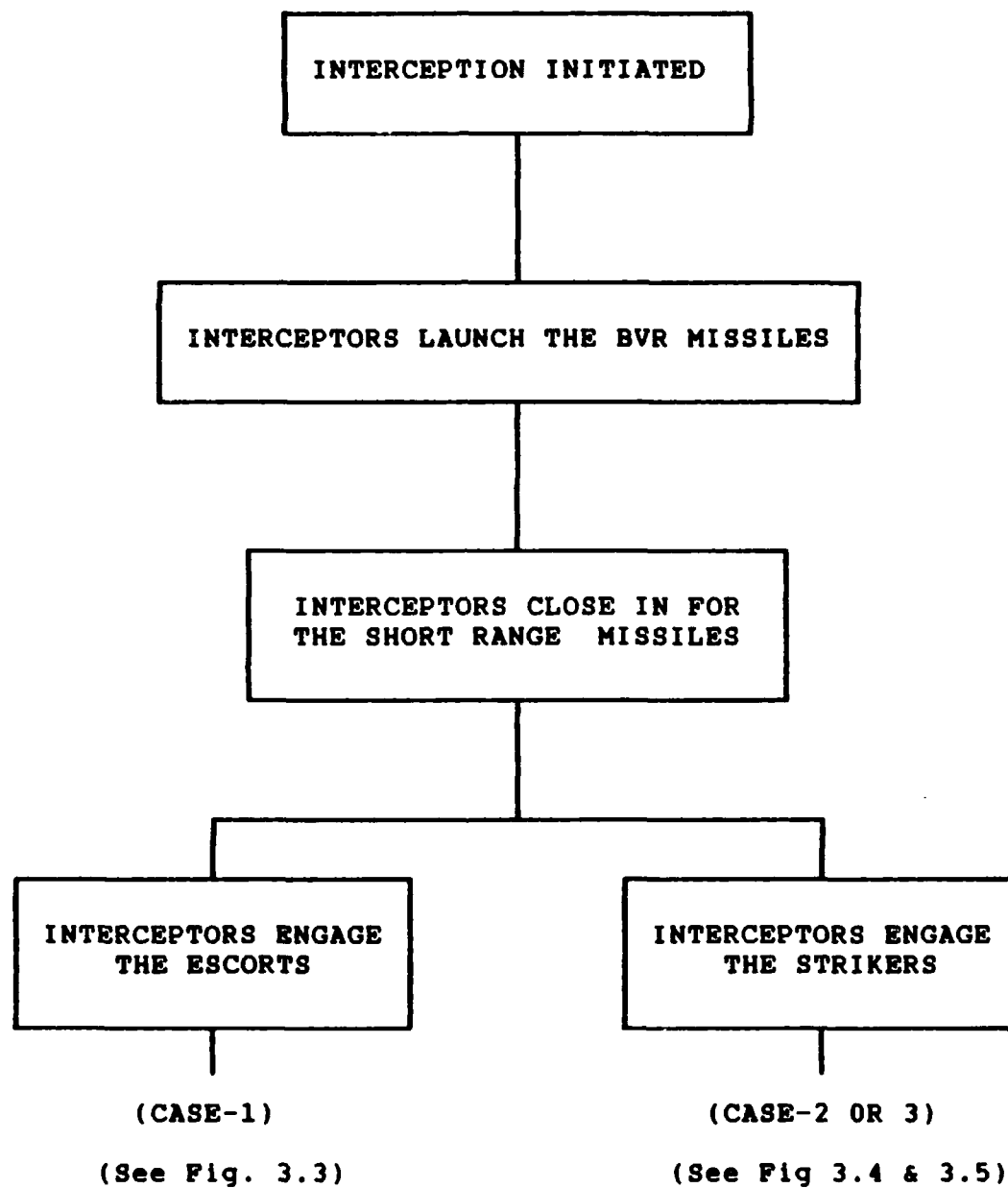


FIGURE 3.2 HIERARCHY OF EVENTS PRIOR TO A CLOSE RANGE COMBAT

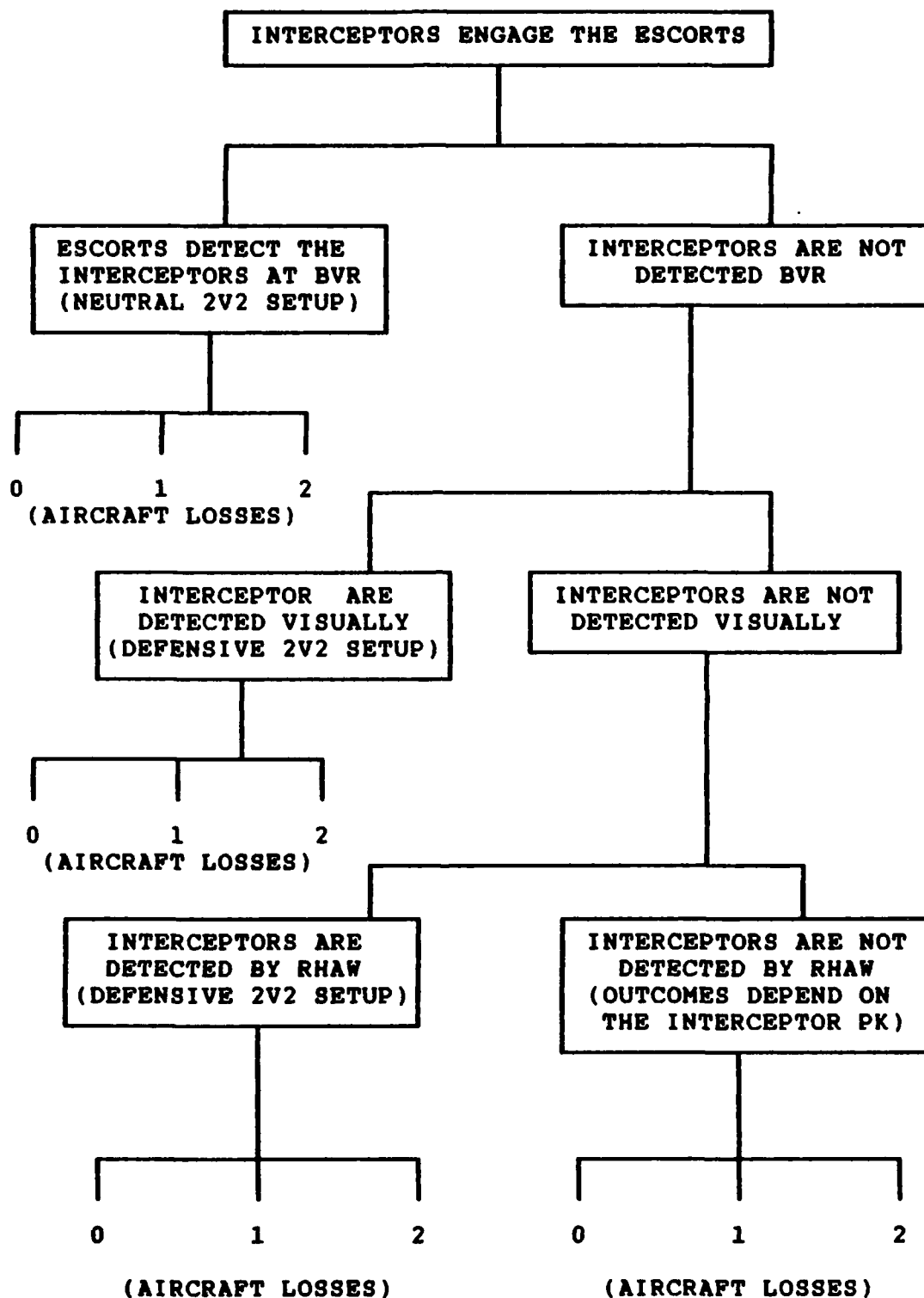


FIGURE 3.3. EVENT HIERARCHY FOR CASE - 1

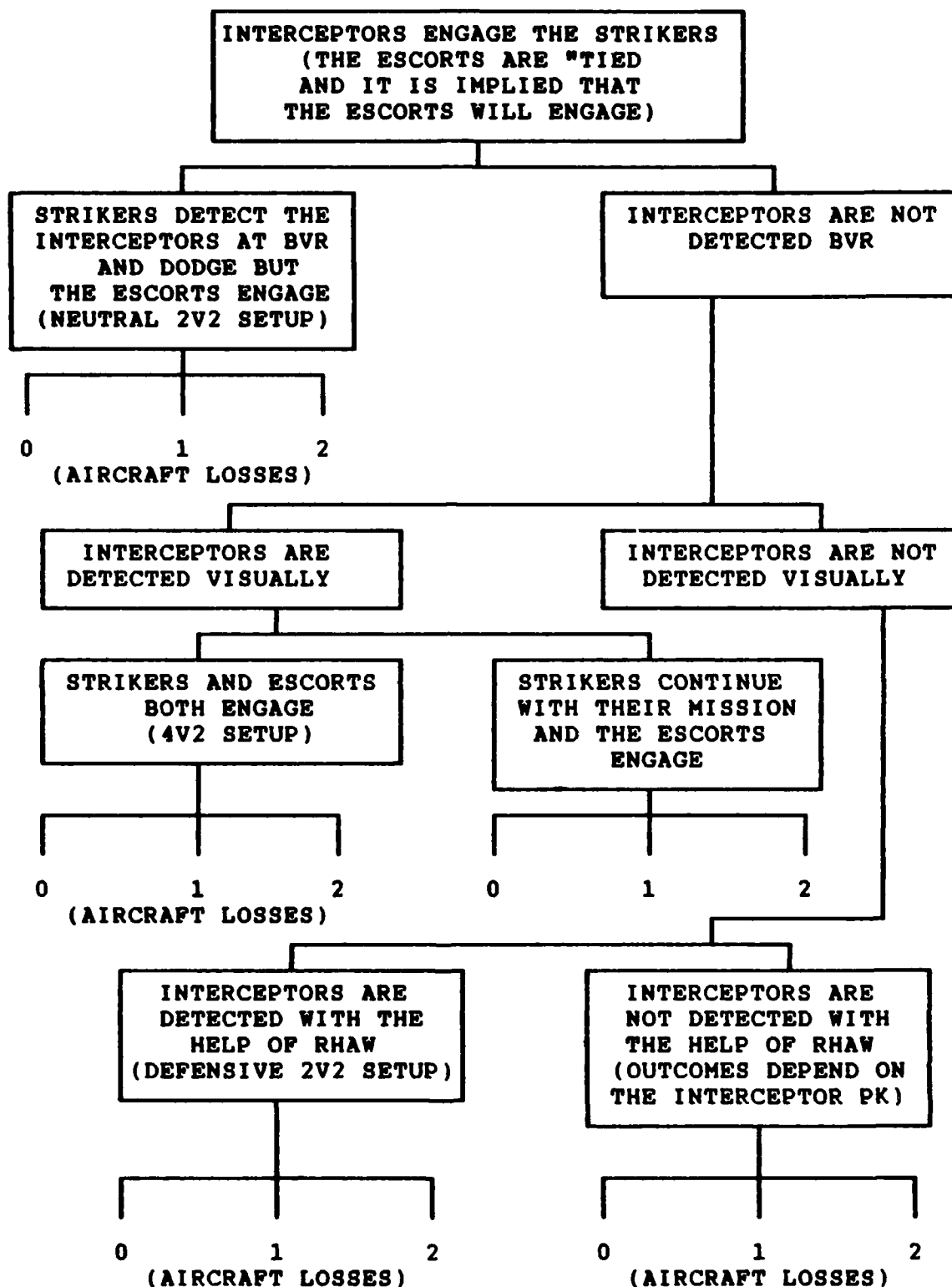


FIGURE 3.4 EVENT HIERARCHY FOR CASE - 2.

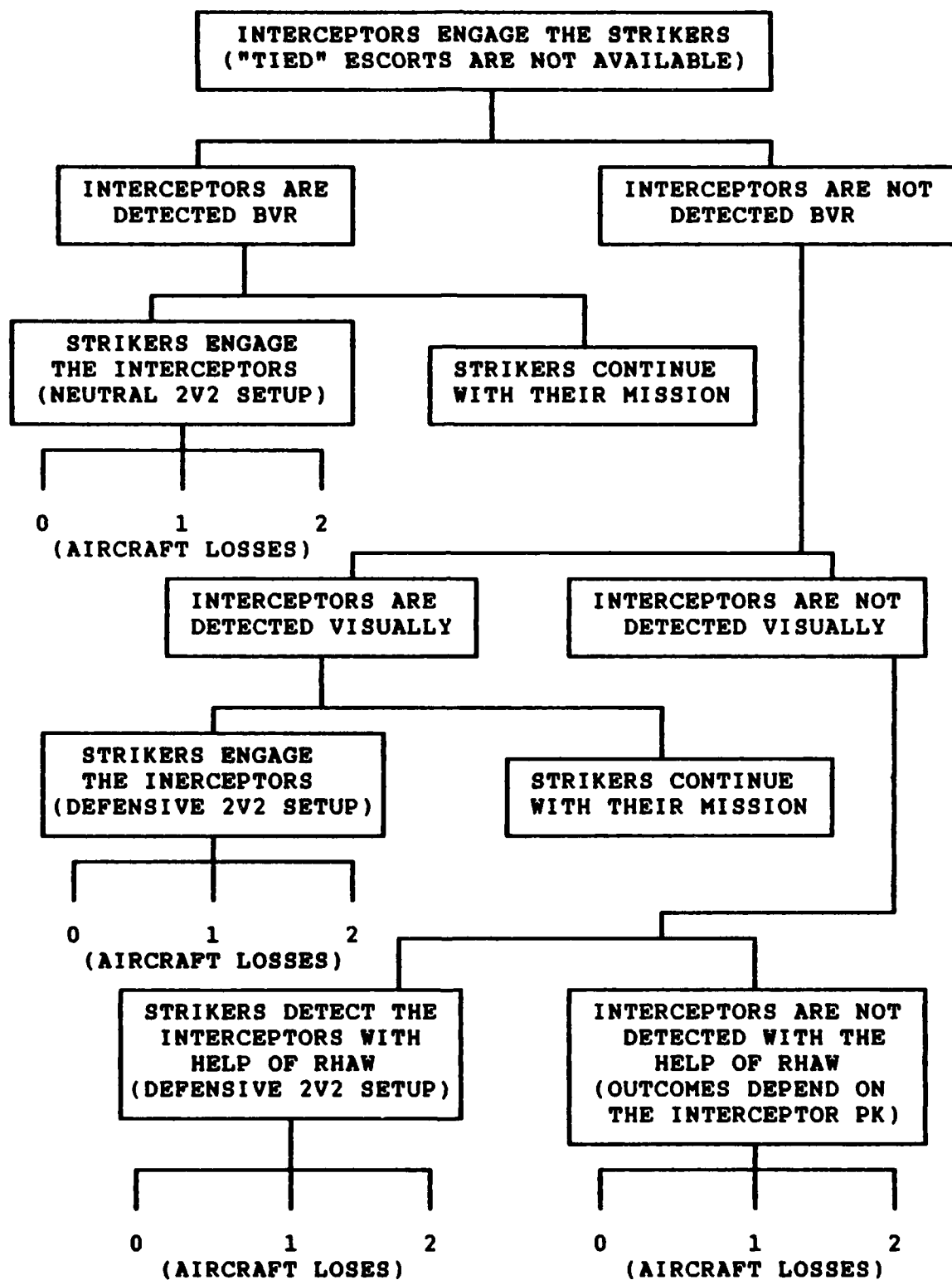


FIGURE 3.5 EVENT HIERARCHY FOR CASE - 3.

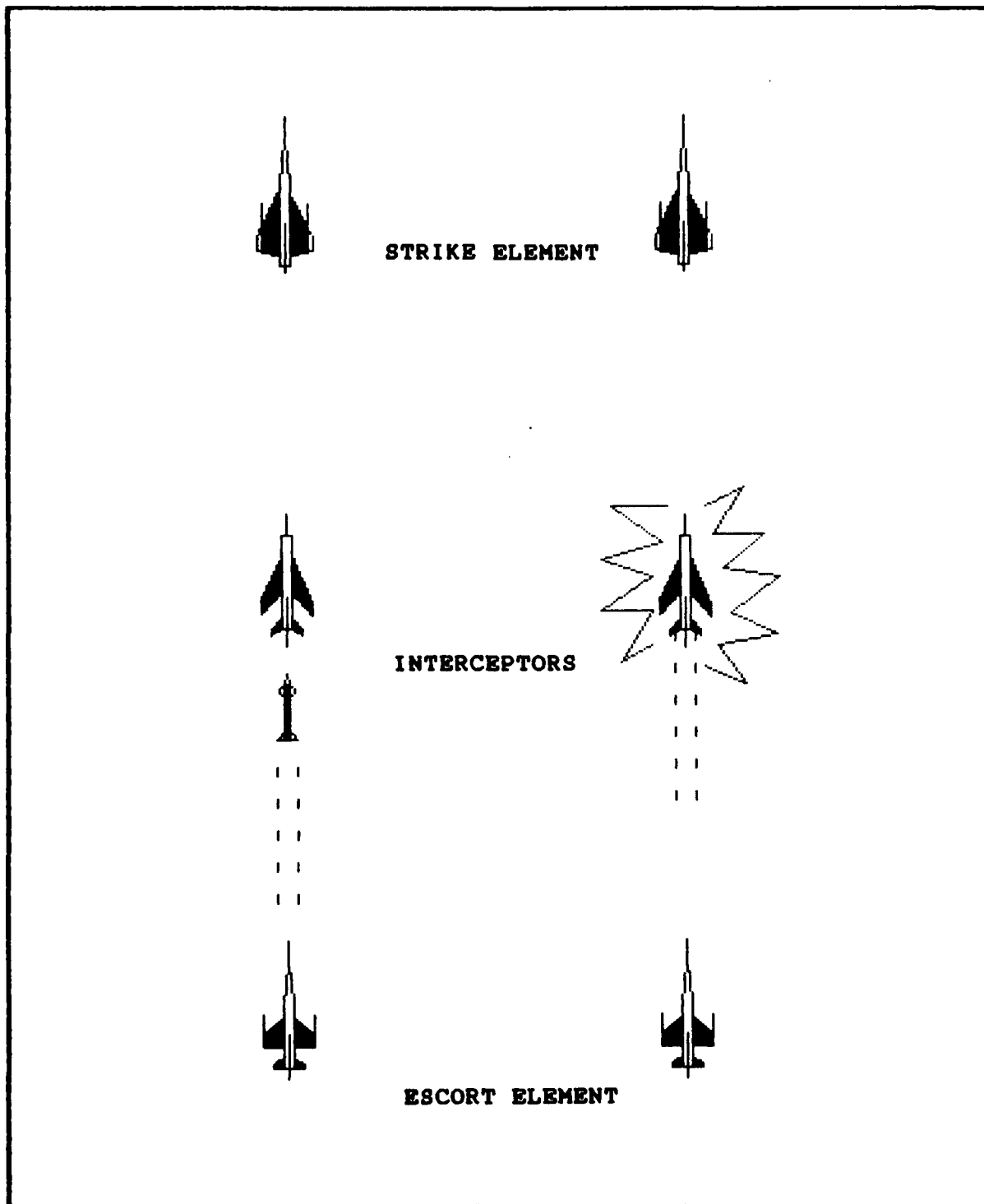


FIGURE 3.6 FORMATION LAYOUT WITH ONE ESCORT ELEMENT AND ONE STRIKE ELEMENT (INTERCEPTORS ATTACKING THE STRIKERS)

visual range, the interceptors either engage the escorts or the strikers but not both at the same time. The decision by the enemy to engage a particular element depends on the formation layout and the positioning of the escorts with respect to the strikers. For example, consider a raider formation with one escort element and one strike element (see Fig. 3.6). The escort element is positioned behind the strike element. In this situation, an attack on the front element will make the interceptors vulnerable to an attack from the rear element. Therefore, the interceptors in most cases will engage the rear element--the element of escort aircraft.

With "free" escorts, the raiders will be divided in two different formations. The probability of the interceptors engaging a particular element will depend upon whether the escorts or the strikers appear first on the enemy radar.

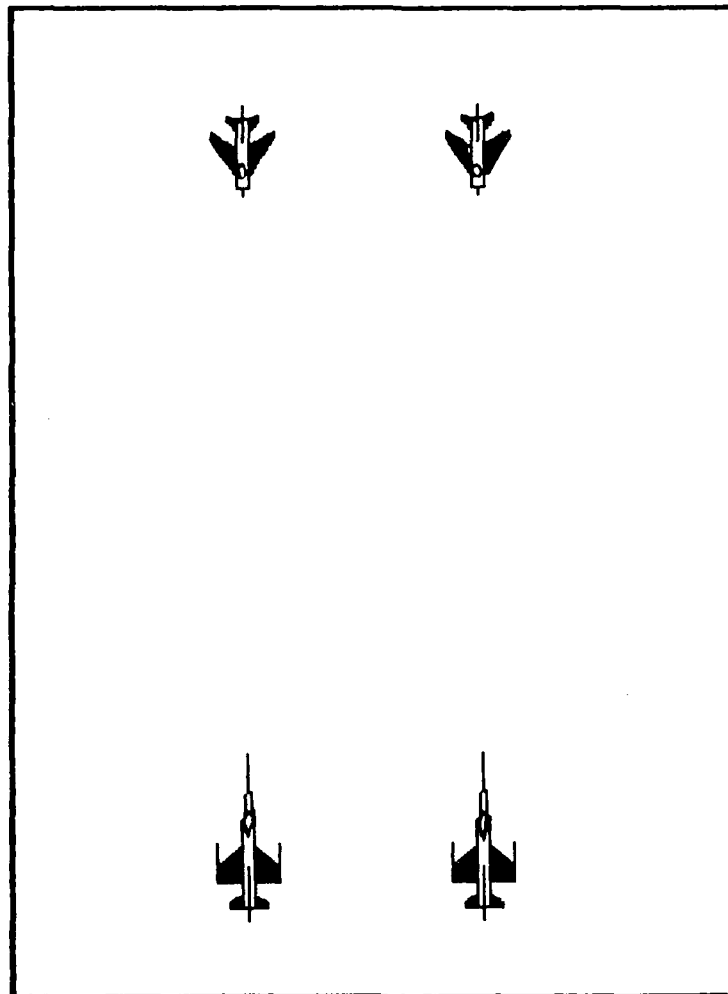
Interception against the Escorts. If the enemy decides to engage an escort element then one of the following events must occur:

- (a) The escorts detect the interceptors at BVR, with the help of their radar.
- (b) The escorts detect the interceptors visually, in the absence of a radar contact.
- (c) A warning is provided by the RHAW system, in the absence of a visual contact.
- (d) The interceptors manage to reach the firing parameters without having been detected.

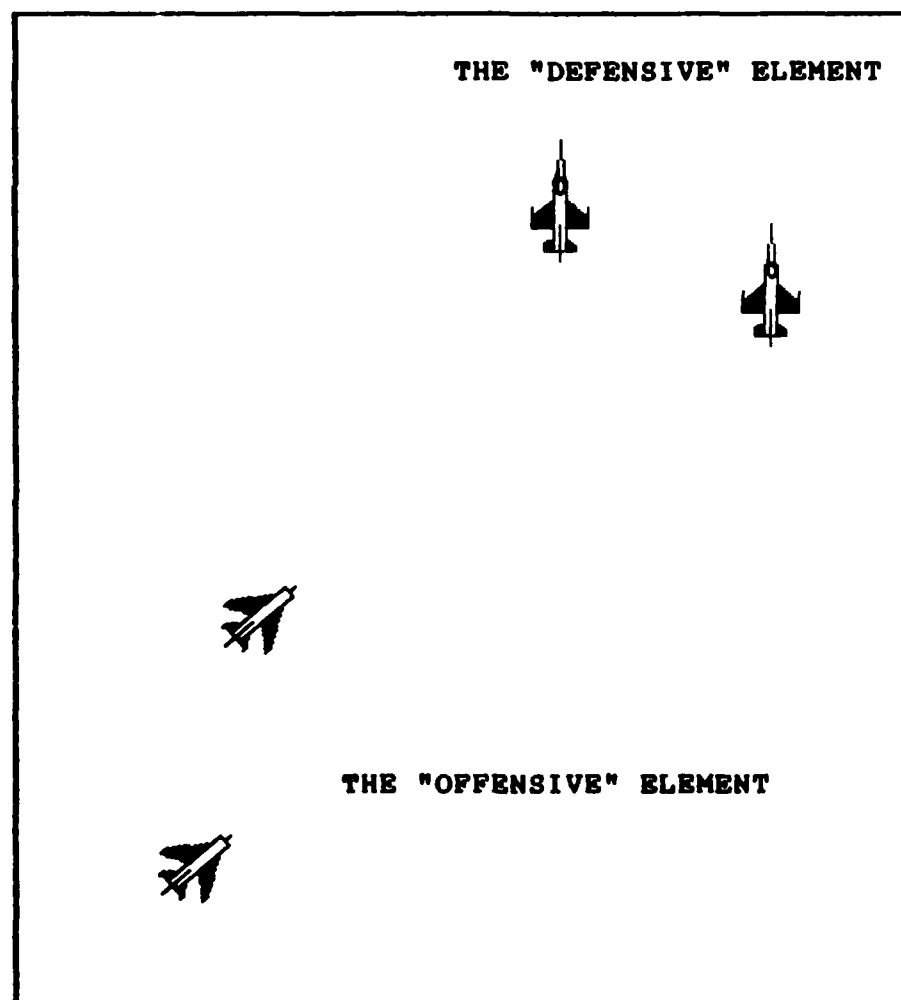
Any of these events will lead to an air combat, in which a compatible number of the escorts will participate and the remaining will proceed with the main formation. It can be assumed that in all cases where the interceptors are detected prior to an attack, only one escort element will participate, and in case of a surprise attack, two escort elements will participate because the element under attack will need help from the next available element. It can also be assumed that the engaged elements will not be able to rejoin with the parent formation because of the displacement created between the two.

Air Combat between two elements. An air combat between two elements is a complex process with innumerable variations. It is a continuous process of situation assessment and the selection of the best course of action. An important decisive factor in this respect is the positional advantage of one element with respect to the other at the beginning of the combat. Figures 3.7 and 3.8 depict a few possible situations for equal or unequal positional advantages. These initial conditions of engagement can broadly be classified as:

- (a) "Neutral" where none of the elements has any advantage over the other.
- (b) "Offensive" where one element has to turn through a smaller angle to point its weapons as compared to the other.
- (c) "Defensive" a state that is opposite to "offensive."



**FIGURE 3.7 TWO ELEMENTS OF AIRCRAFT AGAINST EACH OTHER
IN A "NEUTRAL" SETUP**



**FIGURE 3.8 TWO ELEMENTS OF AIRCRAFT AGAINST EACH OTHER
ONE "DEFENSIVE" AND THE OTHER "OFFENSIVE"**

Regardless of the complexity of the air combat, the possible outcomes of an air combat can be classified as follow:

- (a) Both the pairs escape without any loss.
- (b) One pair loses one and the second pair has no loss.
- (c) Each pair loses one.
- (d) One pair loses two and the other loses one.
- (e) One pair loses two and the other loses none.

(A damaged aircraft can be classified as "escaped" or "destroyed," depending upon whether it recovers back at a home base or not.)

If the enemy losses are disregarded, then the possible outcomes of interest reduce to the following:

- (a) The friendly pair escapes without any loss.
- (b) One friendly aircraft is lost.
- (c) Both the friendly aircraft are lost.

These outcomes are dependent on three main factors: the initial conditions of engagement, the aircraft performance and the pilot ability. Assuming that in a random engagement the pilot ability is the same on both sides, the outcomes mainly depend upon the aircraft performance and the initial conditions of engagement.

For a given set of initial conditions and for a given type of aircraft on each side, the probabilities of the outcomes can be estimated through training missions or by a simulation model.

BVR detection of the Interceptors. The probability of detecting the interceptors at BVR depends on the reliability of

the on-board radar system. The engagement followed by a BVR detection can be assumed to start from a "neutral" setup because the opponents have an equal advantage.

Visual Detection of the Interceptors. In the absence of a radar contact the probability of visually detecting the interceptors depends on the pilot ability. An air combat followed by a visual detection, in most cases, will start from a defensive setup because prior to a visual contact the interceptors are likely to close in to a threatening position.

Detection with the RHAW. A radar warning device, depending upon its reliability, may detect an emission from an interceptor's radar. Detection most likely would occur before a weapons release when the interceptor may lock its radar to the target. The probability of the threat detection, in this case, is the reliability of the RHAW system against that threat. An engagement followed by this event will certainly initiate from a "defensive" setup.

No detection of the interceptors prior to an attack. There exists a possibility that the interceptors may not be detected at all by the target element prior to an attack. A successful missile launch by the interceptors, in this case, will only occur if the exact firing parameters are met. An error on part of the controllers or the interceptor pilots may turn it into an unsuccessful interception. In any case the probability of the first aircraft in the target element to be destroyed will depend on the probability of interceptors positioning behind the target

and the probability of kill of the missile launched. After the release of the first missile, the element of surprise will be lost and the other member of the target element will take an evasive action. Another escort element, if available, will intervene and a combat will initiate.

Interception against the strike element. If the enemy decides to engage one of the strike elements then the situation will be slightly different. A strike element will always endeavor to "escape" rather than "engage." Firstly because its aim will be to reach the target and secondly because its engagement may cause an unnecessary loss. If the threatened strike element detects the interceptors at BVR, an accompanying escort element would also have done the same. The escort element will then intervene and the strike element may be able to escape. In a worst case situation, the strike element may have to jettison its load and may be required to engage along with the escort element. If the formation is un-escorted then the strike element will be required to fight without any intervention of the rest because the primary aim of the mission will be to reach the target.

SUBSEQUENT INTERCEPTIONS

The subsequent interceptions will repeat the same process, except that only those aircraft from the raider formation will participate in the combat which survive the previous interceptions and are able to continue with the mission. It will be appropriate to assume that none of the remaining interceptions will be aimed on an already engaged element because the enemy

will be more concerned about the raiders headed for the target.

The number of interceptions against a particular raid can be estimated by assessing the available number of interceptors on the hostile side, the efficiency of the enemy air defense system, and the number of total raids in progress.

CONCLUSION

The discussion concludes that the air combat between the raiders and the interceptors can be modeled as a series of events, where each event has a certain probability of occurrence. The outcomes of an event are mutually exclusive and their probabilities can be estimated as apriori.

The analysis is based on the following assumptions:

- (a) The interceptor aircraft will fly as pairs and multiple pairs will be used against a large size raid.
- (b) The aircraft from the offensive formation will participate in a combat as elements and not as individuals.
- (c) The surviving aircraft from an engagement will not be able to rejoin with the parent formation.

IV. THE STOCHASTIC MODEL

INTRODUCTION

The previous chapter demonstrates that an air combat between the interceptors and the offensive formation can be broken down into a series of relatively simple events, where each event has a certain probability of occurrence. The combat between the two can, therefore, be modelled as a stochastic process. This chapter describes the application of this concept for the theoretical development of the model. The discussion mainly involves incorporation of two major effects: the effect of a BVR missile attack; and the effect of a close range combat.

THE REQUIRED OUTPUT FROM THE MODEL

The purpose of developing this model is to calculate the following numbers for a given mix of escort and strike elements.

- (a) The expected number of strike and escort aircraft which can continue with the mission after an interception.
- (b) The complement of "(a)", which is the expected number of strike and escort aircraft destroyed during an interception.

THE MARKOV PROCESS

An engagement of the offensive formation with the interceptors may reduce the size of the formation because the aircraft may be destroyed or may have to return to a home base. The number of the two types of aircraft in the offensive formation will, therefore, vary as a function of engagements.

Defining the number of the two types of aircraft as a state, the process of engagements can be modelled as a "finite state Markov Chain."

THE STATE SPACE

The "state space" for the process is the possible combinations of the escort and the strike aircraft which exist at a particular point in time, during the mission.

THE PARAMETER SPACE

The "parameter space" for the process is the number of engagements between the offensive formation and the interceptors.

The engagements can be further classified as:

- (a) A BVR missiles attack on the offensive formation by the interceptors.
- (b) A close-range combat between the offensive formation and the interceptors.

Since the two events will take place in series with certain time interval, they can be assumed to be independent of each other and can be considered as independent transitions of the process.

STATE SPACE FOR A REALISTIC MISSION SCENARIO.

A single offensive formation normally consists of four to eight aircraft. Although The number of aircraft may be higher or lower. Assuming that a maximum number for the aircraft in an offensive formation is ten, the possible states for the process are listed in Table 4.1.

POSSIBLE STATES WITH 10 AIRCRAFT AT THE BEGINNING
(NUMBER OF ESCORT AIRCRAFT, NUMBER OF STRIKE AIRCRAFT)

1	(0,10)	23	(1,7)	45	(6,0)
2	(1,9)	24	(2,6)	46	(0,5)
3	(2,8)	25	(3,5)	47	(1,4)
4	(3,7)	26	(4,4)	48	(2,3)
5	(4,6)	27	(5,3)	49	(3,2)
6	(5,5)	28	(6,2)	50	(4,1)
7	(6,4)	29	(7,1)	51	(5,0)
8	(7,3)	30	(8,0)	52	(0,4)
9	(8,2)	31	(0,7)	53	(1,3)
10	(9,1)	32	(1,6)	54	(2,2)
11	(10,0)	33	(2,5)	55	(3,1)
12	(0,9)	34	(3,4)	56	(4,0)
13	(1,8)	35	(4,3)	57	(0,3)
14	(2,7)	36	(5,2)	58	(1,2)
15	(3,6)	37	(6,1)	59	(2,1)
16	(4,5)	38	(7,0)	60	(3,0)
17	(5,4)	39	(0,6)	61	(0,2)
18	(6,3)	40	(1,5)	62	(1,1)
19	(7,2)	41	(2,4)	63	(2,0)
20	(8,1)	42	(3,3)	64	(0,1)
21	(9,0)	43	(4,2)	65	(1,0)
22	(0,8)	44	(5,1)	66	(0,0)

TABLE 4.1 POSSIBLE STATES WITH A TOTAL OF 10 AIRCRAFT

THE EFFECT OF A BVR MISSILE ATTACK

The BVR missiles are launched with the help of an on-board radar system. Since the shooting aircraft does not have any means of distinguishing between the different types of aircraft, the interceptor aircraft (equipped with the BVR missiles) will select a target at random from the offensive formation. Therefore, each member of the offensive formation will have an equal probability of being shot at.

Suppose the total number of aircraft in the offensive formation is "N", out of which "EN" are the escorts and "SN" are the strikers (EN+SN=N). Also suppose that "BN" missiles can be launched by the interceptors and "PKB" is the probability of kill of each missile.

If all the missiles are fired on different targets, then the number of targets destroyed, has a binomial probability distribution. The probability of "Y" number of targets destroyed, is then given by:

$$P(Y) = \frac{BN!}{Y!(BN-Y)!} \cdot P_{KB}^Y \cdot (1-P_{KB})^{(BN-Y)}$$

The expected number of target aircraft destroyed is given by
(Number of BVR missiles launched).(PK of the individual missile)
which is P_{KB}

$$(BN \cdot P_{KB})$$

the expected number of escorts destroyed is

$$(BN \cdot P_{KB}) \cdot (EN/N)$$

where (EN/N) is the proportion of the escorts in the formation.

Similarly, the expected number of strikers destroyed is

$$(BN.P_{KB}).(SN/N)$$

If the number of BVR missiles that can be launched is more than the targets then the enemy has certain options:

- (a) The interceptors launch only one missile per target
- (b) The interceptors launch more than one missile on certain targets.

In the first case

$$BN = N$$

In the second case there are again two possibilities: either the interceptors launch two missiles per target, one after the other, to increase the probability of destruction, or they wait to see the outcome of the first launch before firing the remaining missiles. The former case is not very likely because it involves an expanded use of expensive missiles. If, however, the enemy is expected to use two missiles per target then the probability of kill of those two missiles can be calculated from PKB , which is given as

$$1 - (1 - PKB)^2$$

and can be used instead of PKB .

In the later case, the interceptors may not get time to launch the second wave of missiles as the rate of closure to the target will be appreciable. If the interceptor aircraft have a capability of launching a missile at distant range where a follow on attack can be made, then the effect of that follow on attack can be incorporated in the model as an independent

transition.

For the purpose of this model, it will be assumed that the interceptors will launch only one BVR missile per target and will make only one BVR missile attack.

THE "ONE STEP" TRANSITION MATRIX FOR THE EFFECT OF BVR ATTACK

Let the one step transition matrix for the effect of BVR missiles be [B], and the probability of going from a state "i" to a state "j" be represented as B_{ij} . Then

$$B_{ij} = 0$$

for all transitions which are impossible.

The probability of a particular possible transition is simply the probability of occurrence of the associated event--the destruction of a specific number of escort and strike aircraft.

The "event" of destroying a specific number of escorts "a" and strike aircraft "b" is conditioned on N, EN, SN, PKB and BN, and will occur when the following conditions are met:

(a) Only "(a+b)" aircraft out of "N" are destroyed by "BN" missiles.

(b) Exactly "a" aircraft are destroyed out of "EN" escorts and "b" aircraft are destroyed out of "SN" strikers.

The probability of destroying "(a+b)" aircraft out of "BN", using the Binomial probability function is given by

$$BIN P(Y=a+b \text{ out of } BN) = \frac{BN!}{(a+b)! (BN-a-b)!} \cdot PKB^{a+b} \cdot (1-PKB)^{BN-a-b}$$

Probability of choosing "a" escort aircraft and "b" striker aircraft from a total of "N" aircraft using the Hyper-geometric

probability function is given by

$$HG P(a, b \text{ out of } N) = \frac{(a+b)!}{a!.b!} \cdot \frac{(N-a-b)!}{(EN-a)!. (SN-b)!} \cdot \frac{EN!.SN!}{N!}$$

The probability of destroying "a" escorts and "b" strikers is then given by

$$P(a, b \text{ destroyed}) = [BIN P(a+b \text{ out of } N)] \cdot [HG P(a, b \text{ out of } N)]$$

A possible transition from state "i" to state "j" will correspond to a specific number of escort and strike aircraft destroyed. For example,

$B_{i,j}$, where $i=j$, will correspond to a condition

$$(a = 0 \text{ and } b = 0)$$

and $B_{i,j}$, where the escort aircraft reduce by "one" and the strike aircraft reduce by "none," will correspond to

$$(a = 1 \text{ and } b = 0)$$

and so on.

Example 4.1

Consider a formation of four escort and six strike aircraft (a total of 10 aircraft). The formation is intercepted by a pair of enemy fighters. Each interceptor can fire one BVR missiles and the probability of kill of each missile is .7.

In this case

$$N = 10 \quad EN = 4 \quad SN = 6$$

$$BN = 2 \quad PKB = .7$$

expected number of total aircraft destroyed = $(.7) \cdot (2) = 1.4$

expected number of escorts destroyed = $(.7) \cdot (2) \cdot (4/10) = .56$

expected number of strikers destroyed = $(.7) \cdot (2) \cdot (6/10) = .84$ The

probability distribution for the possible outcomes is shown in Table 4.2.

OUTCOME	a	b	PROBABILITY
1	0	0	.09
2	1	0	.168
3	0	1	.252
4	1	1	.2613
5	2	0	.0653
6	0	2	.1633

(The probability of outcomes where $(a+b) > BN$ is 0)

TABLE 4.2 THE OUTCOME PROBABILITIES FOR A BVR MISSILE ATTACK

The possible states for this example are the same as in Table 4.1. The probability of a particular transition, B_{ij} , that is the probability of going from a state "i" to a state, can be obtained from the outcome probability distribution in Table 4.2.

For example:

$$\begin{aligned} B_{0,0} &= P(a=0, b=0) \\ &= .09 \end{aligned}$$

$$\begin{aligned} B_{0,1} &= P(a=1, b=0) \\ &= .168 \end{aligned}$$

$$B_{0,10} = P(a=0, b=1)$$

$$= .252$$

$$B_{3,16} = P(a=1, b=1)$$

$$= .2613$$

$$B_{3,17} = P(a=2, b=0)$$

$$= .0653$$

$$B_{3,18} = P(a=0, b=2)$$

$$= .1633$$

All other transitions will have a zero probability in this case.

TRANSITION PROBABILITIES WITH "FREE" ESCORTS

If the escorts are free, then the interceptors will acquire either the escorts or the strikers on their radar but not both. Let "PF" be the probability that the interceptors acquire the escorts. The probability of destroying "a" escorts is conditioned on PKB, BN and PF, and is given by

$$PF \cdot \frac{BN!}{a!(BN-a)!} \cdot PKB^a \cdot (1-PKB)^{(BN-a)}$$

Similarly the probability of destroying "b" strikers is given by

$$(1-PF) \cdot \frac{BN!}{b!(BN-b)!} \cdot PKB^b \cdot (1-PKB)^{(BN-b)}$$

After calculating the outcome probabilities, the same procedure as for the tied escorts, can be followed to find the transition probabilities.

The expected number of escort aircraft destroyed, in this case, is given by

$$PF \cdot PKB \cdot BN$$

and the expected number of strikers destroyed is given by

$$(1-P_r) \cdot P_{KB} \cdot BN$$

ACCOUNTING FOR THE SINGLE AIRCRAFT LEFT OVER

After a BVR missiles attack, the formation will be rearranged to keep the aircraft in elements of two. An odd aircraft will join an existing element to get a cross cover, however, its air to air combat effectiveness will be far less than ideal. In the absence of a supporting element the single aircraft will have to exit the combat area. In case, the single aircraft finds an element to join then for the purpose of the model following assumptions are made:

- (a) If an escort air craft is singled out and if another escort element is available then it will join that escort element, but the combat effectiveness of that element will be assumed to be same as before (a simplifying but safe assumption).
- (b) If more than one, escort elements are available in situation "(a)", then it will join the last escort element to take part in any combat.
- (c) If a single escort aircraft does not find any escort element then it will join a strike element, and in the subsequent combat a pair made out of one escort aircraft and one strike aircraft will participate.
- (d) If a strike aircraft is singled out then it will join a strike element under similar conditions as specified in "(a)" and "(b)".

- (e) If one escort aircraft is singled out along with one strike aircraft, then the two will join to form an element.
- (f) If only one strike aircraft is left then the mission will be aborted regardless of the number of escort aircraft available.

THE EFFECT OF CLOSE-RANGE COMBAT

In chapter III, two important assumptions were made. First it was assumed that in a close combat, the aircraft will participate as elements and not as individuals, and second, the surviving aircraft from a combat engagement will not be able to rejoin with the parent formation. These assumptions imply that the changes in the status of the offensive formation would be "reductions" by elements. For example:

- (a) A reduction by one escort element.
- (b) A reduction by one strike element.
- (c) A reduction by one escort and one strike element etc.

Using the notation " $T_{\alpha\beta}$ " for the transitions, where "E" and "S" represent the number of escort and strike elements by which the formation reduces, the transitions can be classified as

$T_{00}, T_{10}, T_{01}, T_{11}, T_{20}, T_{02}, \dots, T_{\alpha\beta}, \dots$

Further, in Chapter III, it was demonstrated that the process of air combat between the interceptors and the offensive formation can be represented as a "hierarchy of events", where each event has a certain probability of occurrence. The reductions in the offensive formation are directly related to the events of the air combat. For instance, if an element of

escort or strike aircraft engages the interceptors, the formation reduces by one element. The probability of a particular transition is thus the probability of occurrence of the corresponding event of the air combat.

The event hierarchy is shown in figures 4.1 to 4.4. Figure 4.1 shows the initial events of an interception, and the rest of the figures, 4.2 to 4.4, correspond to three distinct cases of the subsequent events, which are as follows:

Case-1 An engagement between the interceptors and the escort aircraft (Fig. 4.2).

Case-2. An engagement between the interceptors and the strike aircraft which implies that the escorts will also intervene (Fig. 4.3).

Case-3. An engagement between the interceptors and the strike aircraft in the absence of escort aircraft (Fig. 4.4).

LIST OF THE SYMBOLS USED IN THE EVENT HIERARCHY

The symbols used in the hierarchy are defined as follows:

P_r - The probability that the interceptors are directed to one of the escort elements.

P_{BVR_E} - The probability of detecting the interceptors with the on-board radar by the escort aircraft at BVR.

P_{BVR_S} - The probability of detecting the interceptors with the on-board radar by the strike aircraft at BVR.

P_{VIS} - The probability of visually detecting the interceptors.

P_{PRHAWK} - The probability of detecting the interceptors with

the help of RHAW (Radar Warning And Homing system), by the escort aircraft.

P_{RHAW} - The probability of detecting the interceptors with the help of RHAW, by the strike aircraft.

P_{DVT} - The probability that a strike element can dodge the interceptors after detecting them visually, and can continue with the mission, given that the escorts are tied.

P_{DVF} - The probability that a strike element can dodge the interceptors after detecting them visually, and can continue with the mission, given that the escorts are free.

P_{DR} - The probability that the strike element can dodge the interceptors after detecting them with the radar, given that the escorts are free.

$P_{EN}(k)$ - The probability of "k" number of escort aircraft being destroyed in a neutral setup against the interceptors.

$P_{SN}(k)$ - The probability of "k" number of strike aircraft being destroyed in a neutral setup against the interceptors.

$P_{ED}(k)$ - The probability of "k" number of escort aircraft being destroyed in a defensive setup against the interceptors.

$P_{SD}(k)$ - The probability of "k" number of strike aircraft being destroyed in a defensive setup against the interceptors.

$P_{4v2}(k)$ - The probability of "k" number of strike aircraft being destroyed in a 4-verses-2 setup against the interceptors.

$P_K(k)$ - The probability of "k" aircraft being destroyed, given that the interceptor's attack was undetected.

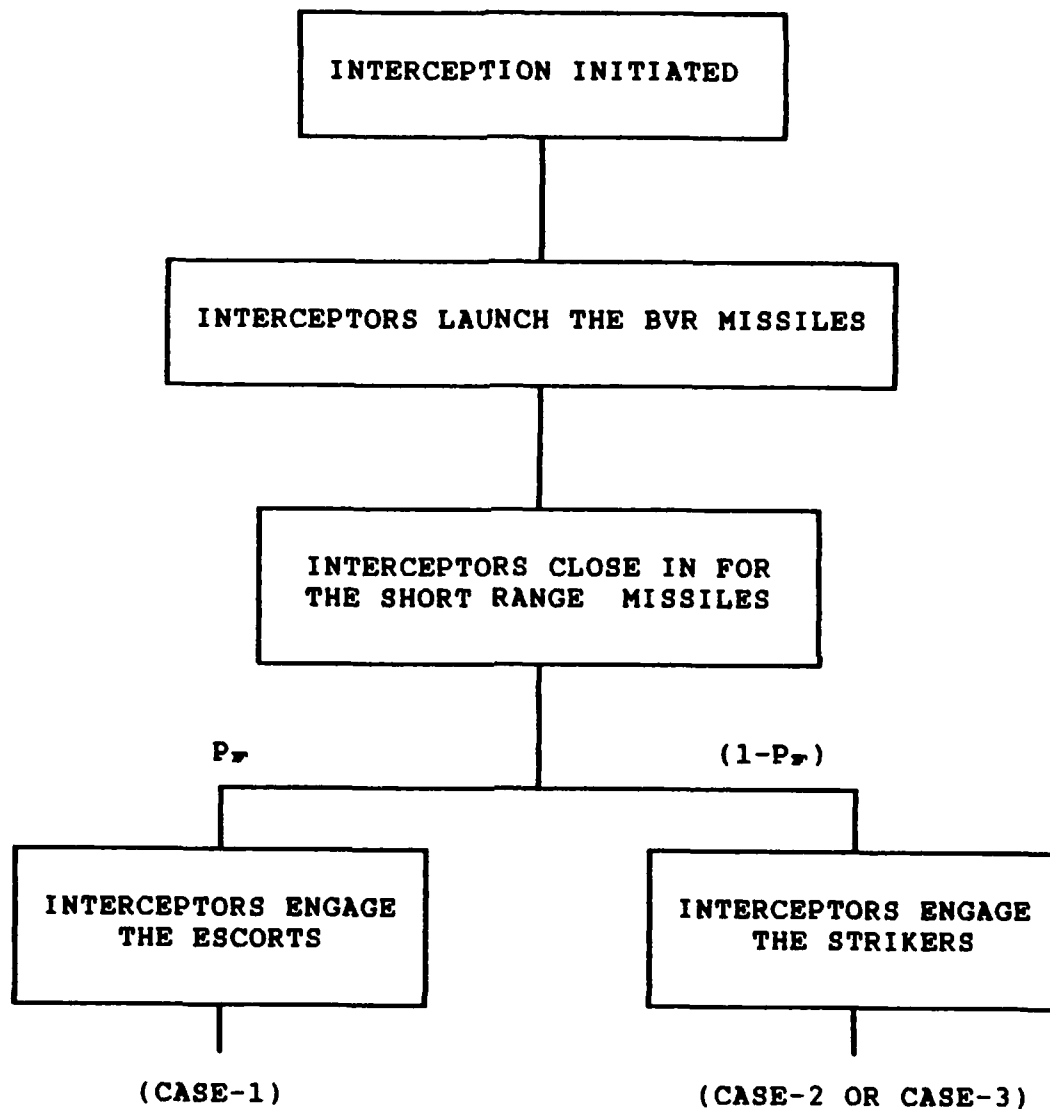


FIGURE 4.1 HIERARCHY OF EVENTS PRIOR TO A CLOSE RANGE COMBAT

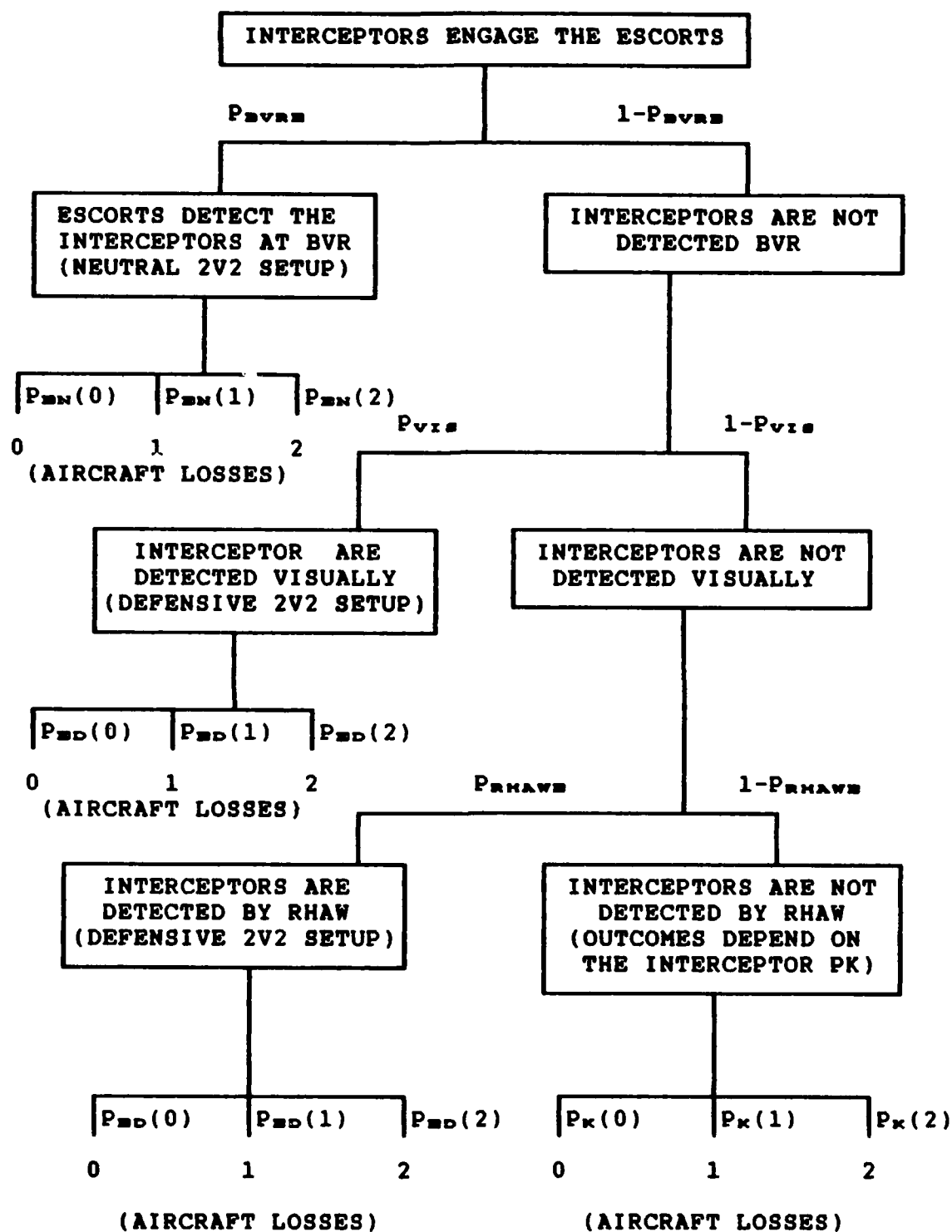


FIGURE 4.2 EVENT HIERARCHY FOR CASE - 1

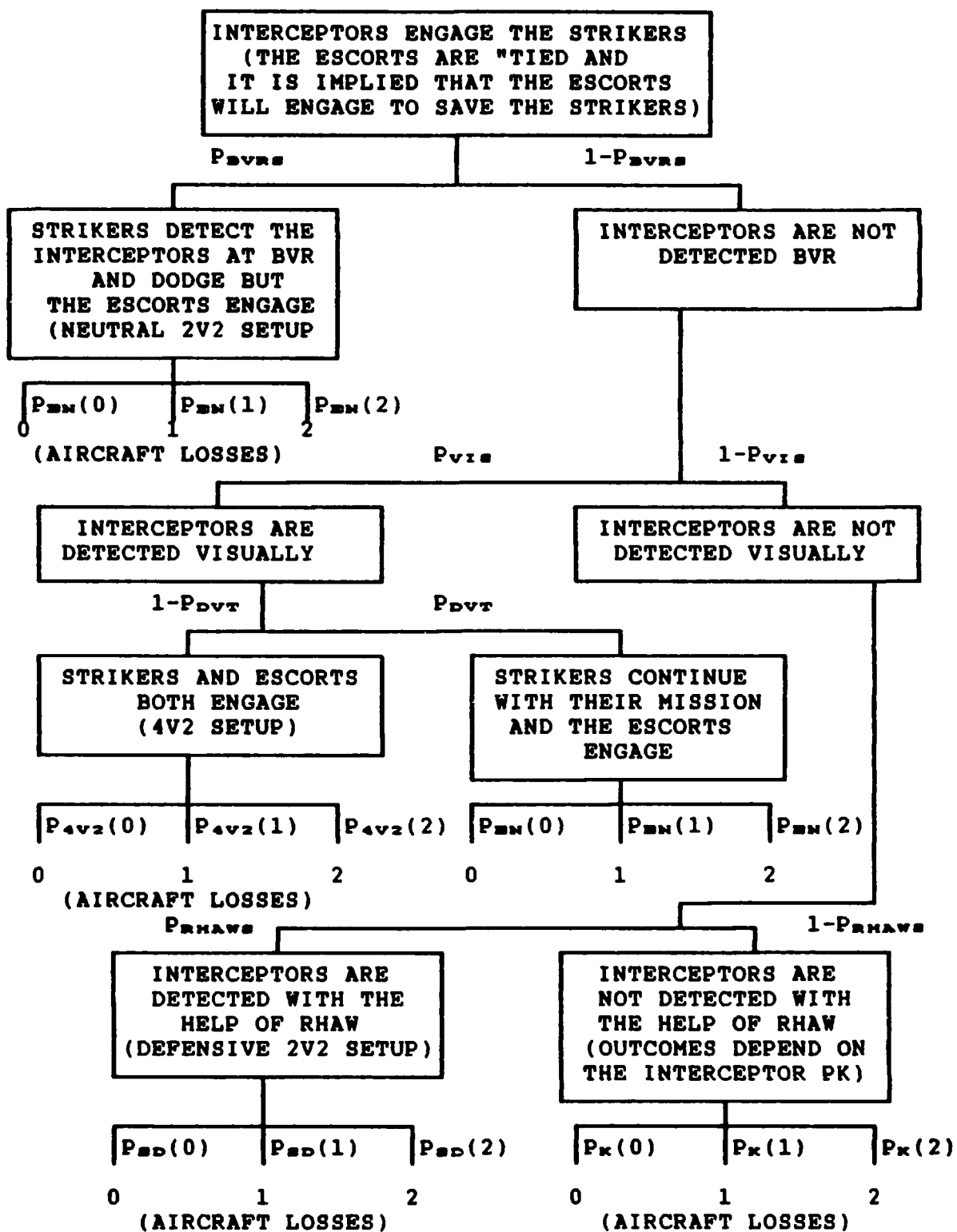


FIGURE 4.3 EVENT HIERARCHY FOR CASE - 2.

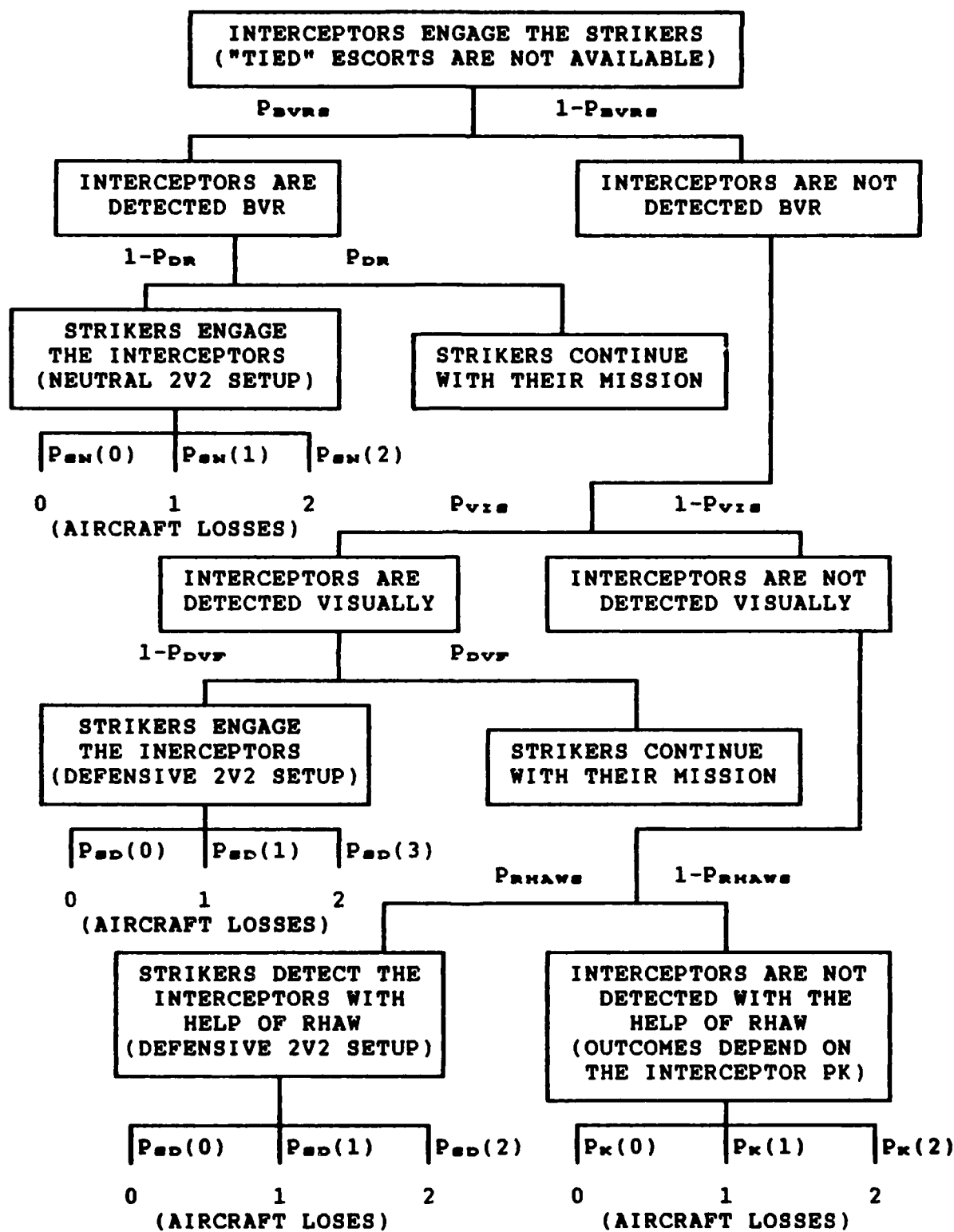


FIGURE 4.4 EVENT HIERARCHY FOR CASE - 3.

TRANSITION PROBABILITIES

The transition probabilities can be calculated from the event probabilities. There are no events on the hierarchy which imply a reduction of the offensive formation by more than two elements. Therefore, the only probabilities that need to be calculated are, for the reductions by two or less than two elements.

The probability of transition-"T₂₀". This transition will only occur if the escort force has two or more elements and both of them engage in a combat with the interceptors. From the event-hierarchy we find that such an event is expected if:

- (a) the interceptors engage the escorts, and
- (b) the interceptors reach the firing parameters without having been detected.

The probability of this event (as calculated from the event hierarchy), is given by

$$P(T_{20}) = (1-P_{RHAWE}) \cdot (1-P_{VIB}) \cdot (1-P_{BURE}) \cdot P_F \quad (4.1)$$

The probability of transition-"T₁₀". Transition "T₁₀" will occur if there is at least one element of escort aircraft present which engages the interceptors. This transition can occur under various situations, which need to be considered individually.

- (a) The interceptors engage the escorts and there are more than one elements of escort aircraft. The transition "T₁₀" will occur if the interceptors are engaged by only one element of escorts. The probability of "T₁₀" is, therefore, obtained by subtracting the probability of an engagement of

more than one escort element, from the probability of engagement of at least one escort element. which is given by

$$P_a(T_{10}) = P_r - P(T_{20})$$

(b) The interceptors engage the escorts and there is only one escort element present. If there is only one escort element present then a reduction by two escort elements is not possible. Therefore, the probability of "T10" is the same as the probability of an engagement of at least one escort element, which is given by

$$P_b(T_{10}) = P_r$$

(c) The interceptors engage the strikers and the escorts intervene. The responsibility of the escorts is to save the strikers. Therefore, an element of escorts will always engage the interceptors if the interceptors threaten the strikers. In this case, either a transition "T10" or a transition "T11" will occur, depending upon whether the threatened strike element manages to dodge the interceptors or is forced to engage. The probability of transition "T10", in this case, is the probability of

- (i) the interceptors threatening the strikers (given that the escorts are tied), and
- (ii) the strikers successfully evading the interceptors.

Which is given by

$$P_c(T_{10}) = P_{DVRB} \cdot (1 - P_r) + P_{DVT} \cdot P_{VIB} \cdot (1 - P_{DVRB}) \cdot (1 - P_r)$$

The probability of transition "T10", taking into account all

possible cases, is then given by

$$\begin{aligned} P(T_{10}) &= P_a(T_{10}) + P_c(T_{10}) \\ &= P_r - P(T_{20}) + P_{\text{DVRB}}.(1-PF) \\ &\quad + P_{\text{DVT}}.P_{\text{VIB}}.(1-P_{\text{DVRB}}).(1-PF) \end{aligned} \quad (4.2a)$$

(if there are more than one escort elements)

and

$$\begin{aligned} P(T_{10}) &= P_b(T_{10}) + P_c(T_{10}) \\ &= P_r + P_{\text{DVRB}}.(1-P_r) \end{aligned} \quad (4.2b)$$

(if there is only one escort element)

The probability of transition-"T₂₀" with free escorts. On the event-hierarchy, the event corresponding to a reduction of two escort elements can only occur if the interceptors engage the escorts and not when they engage the strikers. Therefore, the probability of transition "T₂₀" is the same as that for the tied escorts.

The probability of transition-"T₁₀" with free escorts. Since the "free" escorts cannot intervene in an engagement between the interceptors and the strikers, the transition "T₁₀" will only occur if the interceptors engage the escorts, except when "T₂₀" occurs. The probability of transition "T₁₀" in this case is given by

$$P(T_{10}) = P_r - P(T_{20}) \quad (4.2aa)$$

In case there is only one element of "free" escorts then "T₂₀" cannot occur. Instead, "T₁₀" will occur. Therefore, with only one element of free escorts, the probability of transition "T₁₀" is given by

$$P(T_{10}) = P_r \quad (4.2bb)$$

The probability of transition-"T₁₁". As discussed in the last paragraph, a transition "T₁₁" will occur if the interceptors threaten a strike element in the presence of escorts and that strike element is forced to engage. The probability of this event, is given by

$$P(T_{11}) = (1-P_{VIS}).(1-P_{DVRB}).(1-P_r) \\ + (1-P_{DVT}).P_{VIS}.(1-P_{DVRB}).(1-P_r)$$

or

$$P(T_{11}) = (1-P_r).(1-P_{DVRB}).[(1-P_{VIS}) + P_{VIS}.(1-P_{DVT})] \quad (4.3a) \\ \text{(if the escorts are tied)}$$

and

$$P(T_{11}) = 0 \quad \text{(otherwise)} \quad (4.3b)$$

The probability of transition-"T₀₁". A transition "T₀₁" corresponds to a reduction by one strike element only. An event of this nature will only occur if:

- (a) the interceptors engage the strikers, and
- (b) the escorts are either free or are not available at all,

except when:

- (a) the threatened strike element manages to dodge the interceptors, or
- (b) more than one strike elements engage the interceptors.

Therefore, if the escorts are "tied" then

$$P(T_{01}) = 0 \quad (4.4a)$$

And if the escorts are not tied then the probability of

transition "T₀₁" can be obtained by subtracting the probability of an engagement of two strike elements and the strikers probability of evading the interceptors, from the probability of strikers being threatened. which is given by

$$\begin{aligned}
 P(T_{01}) &= P_{RHAWB} \cdot (1 - P_{VIS}) \cdot (1 - P_{SVRS}) \cdot (1 - P_F) \\
 &\quad + (1 - P_{DVF}) \cdot P_{VIS} \cdot (1 - P_{SVRS}) \cdot (1 - P_F) \\
 &\quad + (1 - P_{DR}) \cdot P_{SVRS} \cdot (1 - P_F) \\
 &= (1 - P_F) \cdot \{ (1 - P_{SVRS}) [P_{RHAWB} \cdot (1 - P_{VIS}) + P_{VIS} \cdot (1 - P_{DVF})] \\
 &\quad + P_{SVRS} \cdot (1 - P_{DR}) \} \quad (4.4b)
 \end{aligned}$$

The probability of transition-"T₀₂". Transition "T₀₂" will occur under following conditions:

- (a) the escort aircraft are either "free", or are not available, and
- (b) the interceptors reach the firing parameters behind a strike element without having been detected.

In this situation a non-threatened strike element, if present, will intervene to save the threatened strike element. The probability of this event is given by

$$P(T_{02}) = 0 \quad (\text{if the escorts are tied}) \quad (4.5a)$$

$$P(T_{02}) = (1 - P_{RHAWB}) \cdot (1 - P_{VIS}) \cdot (1 - P_{SVRS}) \cdot P_F \quad (\text{otherwise}) \quad (4.5b)$$

The probability of transition-"T₀₀". Transition "T₀₀" corresponds to "no reduction" in the offensive formation. This event is not expected if the escorts are tied because the escorts will always engage the interceptors if the strike force is threatened. Therefore, if the escorts are "tied" then

$$P(T_{00}) = 0 \quad (4.6a)$$

However, in the absence of escorts, the strikers will endeavor to dodge the interceptors, and if they succeed then a transition "Too" will occur. The probability of transition "Too" is, therefore, the probability of strikers successfully evading the interceptors (in the absence of escorts). Which is given by

$$\begin{aligned} P(T_{00}) &= P_{DR} \cdot P_{EVR} \cdot P_r + P_{DVR} \cdot P_{VIS} \cdot (1 - P_{EVR}) \cdot P_r \\ &= P_r \cdot [P_{DR} \cdot P_{EVR} + P_{DVR} \cdot P_{VIS} \cdot (1 - P_{EVR})] \end{aligned} \quad (4.6b)$$

THE "ONE STEP" TRANSITION MATRIX

Let the number of possible states for the system be "m", and let the "one step" transition matrix for the effect of close air combat be [P]. Then [P] has (m x m) dimensions. Since certain transitions are not possible because the number of elements cannot increase, therefore, the elements of [P], corresponding to such transitions have a zero value. The rest of the elements of [P] correspond to one of the categories of transitions "T_{es}", and the value of those elements can be determined from the corresponding probabilities of the transitions "T_{es}".

EXAMPLE 4.2

Consider a close-range encounter between the offensive formation and a pair of interceptors. Through past experience, the probabilities relating to the combat have been determined as follows:

$$P_r = 0.8$$

$$P_{EVR} = 0.7$$

$$P_{DVR} = 0.6$$

$$P_{VIS} = 0.8$$

$$P_{RHAWK} = 0.6$$

$$P_{RHAWK} = 0.6$$

$$P_{DVT} = 0.7$$

$$P_{DVF} = 0.2$$

$$P_{DR} = 0.6$$

The problem is to calculate the transition probabilities (TES).

Solution

If the escorts were "tied", then

$$P(T_{20}) = (1 - P_{RHAWK}) \cdot (1 - P_{VIS}) \cdot (1 - P_{DVRE}) \cdot P_F$$

$$P(T_{10}) = P_F - P(T_{20}) + P_{DVRE} \cdot (1 - P_F) \\ + P_{DVT} \cdot P_{VIS} \cdot (1 - P_{DVRE}) \cdot (1 - P_F)$$

$$P(T_{11}) = (1 - P_F) \cdot (1 - P_{DVRE}) \cdot [(1 - P_{VIS}) + P_{VIS} \cdot (1 - P_{DVT})]$$

$$P(T_{01}) = 0$$

$$P(T_{02}) = 0$$

$$P(T_{00}) = 0$$

on substituting the given values into these equations, we obtain

$$P(T_{20}) = 0.019$$

$$P(T_{10}) = 0.945$$

$$P(T_{11}) = 0.035$$

$$P(T_{01}) = 0.$$

$$P(T_{02}) = 0.$$

$$P(T_{00}) = 0.$$

if the escorts were "free", then

$$P(T_{20}) = (1 - P_{RHAWK}) \cdot (1 - P_{VIS}) \cdot (1 - P_{DVRE}) \cdot P_F$$

$$P(T_{10}) = P_F - P(T_{20})$$

$$P(T_{11}) = 0$$

$$P(T_{01}) = (1-P_F) \cdot \{ (1-P_{BVR}) [P_{RHAWB} \cdot (1-P_{VIS}) + P_{VIS} \cdot (1-P_{DVF})] + P_{BVR} \cdot (1-P_{DR}) \}$$

$$P(T_{02}) = (1-P_{RHAWB}) \cdot (1-P_{VIS}) \cdot (1-P_{BVR}) \cdot (1-P_F)$$

$$P(T_{00}) = (1-P_F) \cdot \{ P_{DR} \cdot P_{BVR} + P_{DVF} \cdot P_{VIS} \cdot (1-P_{BVR}) \}$$

and on substitution, we get

$$P(T_{20}) = 0.019$$

$$P(T_{10}) = 0.78$$

$$P(T_{11}) = 0.$$

$$P(T_{01}) = 0.109$$

$$P(T_{02}) = 0.0064$$

$$P(T_{00}) = 0.084$$

The probabilities calculated in this example are for "reductions" in the offensive formation. For example,

$$P(T_{10}) = 0.78$$

indicates that the probability of the offensive formation reducing by one escort element and no strike element is 0.78. The "one step" transition matrix [P] can now be constructed from the calculated probabilities.

TRANSITION PROBABILITIES AFTER "n" INTERCEPTIONS

While intercepting the offensive formation, the interceptors will first launch the BVR missiles and will subsequently close in for a close-range encounter. The transition probabilities for one interception, which includes one BVR engagement and one close-range engagement are given by

$$[B] \cdot [P]$$

where [B] is the "one step" transition matrix for the effect of BVR missiles, and [P] is the "one step" transition matrix for the effect of close air combat.

The same process will be repeated for the subsequent interceptions. If the formation is sequentially intercepted by different types of interceptors then the "one step" transition matrix for each engagements will be different. The transition probabilities after "n" interceptions will be given by the product of the "one step" transition matrices for all engagements.

EXPECTED NUMBER OF AIRCRAFT DESTROYED DURING A CLOSE AIR COMBAT

The transitions occur as a result of specific events, and the events lead to outcomes in the form of destruction of friendly aircraft. The outcomes, therefore, correspond to the transitions. A transition may be associated with a destruction of one, or two, or no friendly aircraft. Since the outcomes are probabilistic, an expectation for the outcome can be calculated for a given transition. For example, if the probabilities of "one", "two" and "nil" friendly aircraft destroyed due to an event are: $P(1)$, $P(2)$ and $P(0)$, respectively. Then the expected number of friendly aircraft destroyed for that event is given by

$$P(0).(0) + P(1).(1.0) + P(2).(2.0)$$

The expectation, calculated in this manner, will be conditioned on the occurrence of that particular transition.

Let [R] be a matrix of the same dimensions as [P], and let the elements of [R] be the expected number of aircraft destroyed

associated with a transition from a state "i" to another state "j". Alternately, " P_{ij} " is the probability of a transition from state "i" to "j", and " R_{ij} " is the expected number of aircraft destroyed for the same transition.

The expected number of friendly aircraft destroyed during the close air combat is then given by

$$P_{i1}.R_{i1} + P_{i2}.R_{i2} + P_{i3}.R_{i3} + P_{i4}.R_{i4} \dots\dots + P_{im}.R_{im}$$

where "i" is the state of the system before the combat starts. The expected number of aircraft destroyed in this case is conditioned on the initial state "i" of the system.

Let [D] be a (m x 1) matrix, where D_i is the expected number of aircraft destroyed during the close air combat, given that the system was in a state "i" before the initiation of the combat. Then

$$D_i = P_{i1}.R_{i1} + P_{i2}.R_{i2} + P_{i3}.R_{i3} \dots\dots + P_{im}.R_{im}$$

$$(\text{for } i = 1, 2, 3, \dots, m)$$

THE EXPECTED NUMBER OF AIRCRAFT DESTROYED DURING THE "nth" CLOSE-RANGE ENGAGEMENT

The state of the system will certainly be known at the beginning of the mission, but the system may be found in any of the states ranging from "1" to "m" after one or more engagements. If $[P^{n-1}]$ represents the transition matrix prior to the "nth" engagement then the probability of finding the system in any of the states (ranging from "1" to "m"), before the "nth" engagement is given by the "ith" row of the $[P^{n-1}]$ transition matrix, where "i" is the state at the beginning of the mission. The expected

number of friendly aircraft destroyed during the "nth" engagement is then given by

$$P_{11}^{n-1}.D_1 + P_{12}^{n-1}.D_2 + P_{13}^{n-1}.D_3 + \dots + P_{1m}^{n-1}.D_m$$

THE EXPECTED NUMBER OF AIRCRAFT DESTROYED DURING "n" INTERCEPTIONS

The expected number of friendly aircraft destroyed during "n" interception is the sum of the expected losses for all interceptions, including the BVR as well as close-range encounters.

THE EXPECTED NUMBER OF AIRCRAFT AVAILABLE WITH THE OFFENSIVE FORMATION AFTER "n" INTERCEPTIONS

The "ith" row of the transition matrix $[P^n]$ provides the probabilities of finding the system in any of the resulting states after "n" interceptions, where "i" is the state at the beginning of the mission. If " E_j " is the number of escort aircraft, and " S_j " is the number of strike aircraft in the offensive formation while the formation is in state "j" ($j = 1, 2, 3, \dots, m$), then the expected number of escort aircraft with the formation after "n" interceptions is given by

$$P_{11}^n.E_1 + P_{12}^n.E_2 + P_{13}^n.E_3 + \dots + P_{1m}^n.E_m$$

and the expected number of strike aircraft in the formation is given by

$$P_{11}^n.S_1 + P_{12}^n.S_2 + P_{13}^n.S_3 + \dots + P_{1m}^n.S_m$$

V. COMPUTERIZATION OF THE MODEL

A computer program for the model is written in Fortran-77, and included in the paper as Appendix A. The intent is to demonstrate the practicality of the model, however, it may not be the most efficient program. This chapter describes the salient features of the program. An example is also added to demonstrate the use of the computer model.

INPUT

The user has to estimate the number of interceptions expected enroute and determine the values of the variables: PF, PBVRE, PBVRS, PVIS, PRHAW, PRHWS, PDVT, PDVF, PDR, PKB, BN, PK1, PK2, PEN1, PEN2, PED1, PED2, PSN1, PSN2, PSD1, PSN2, P4V21 and P4V22 (These variables have already been defined in Chapter IV). The values are to be determined for each individual interception and written in separate files, with one file for one interception (a specimen "input file" is shown in Appendix B). The number of the escort and the strike aircraft at the beginning of the mission and the status of the escorts ("free" or "tied") is also to be specified interactively.

OUTPUT

The program provides the following output results:

- (a) The expected number of aircraft destroyed after each interception.
- (b) The expected number of strike and escort aircraft available after each interception.

A specimen output report is shown in Appendix C.

THE PROGRAM STRUCTURE

The program consists of 9 subroutines and 14 functions. The flow diagram of the program is shown in Figure 6.1. The main program collects the input data for one interception at one time sequentially and computes the following by calling the appropriate subroutine:

- (a) The "one step" transition matrix for the effect of BVR missile attack and the "progressive" transition matrix for the mission ("progressive" transition matrix is the product of all the "one step" transition matrices till that time).
- (b) The expected number of the friendly aircraft destroyed during the BVR missile attack and the "accumulated" number of aircraft destroyed till that time.
- (c) The "one step" transition matrix for the effect of close air combat and the "progressive" transition matrix for the mission.
- (d) The expected number of the friendly aircraft destroyed during the close air combat and the "accumulated" number of aircraft destroyed till that time.
- (e) The output results.

The purpose of the individual subroutines and the functions has been included in the program itself, and the intermediate variables have also been defined within the program.

VERIFICATION

The subroutines and the functions were tested individually and produced the same results as with the hand calculations, or as expected.

APPLICATION OF THE MODEL

The following example demonstrates the use of the model. The numbers used in the example are only for the purpose of illustration.

EXAMPLE 5.1a

Consider a situation where a commander of a tactical attack unit has to send a strike mission against an enemy target. A total of eight aircraft are available for the mission, and the commander decides to keep the escorts "tied". The following options are available to configure the aircraft:

1. All aircraft as strikers.
2. 2 aircraft as escorts and 6 as strikers.
3. 4 aircraft as escorts and 4 as strikers.
4. 6 aircraft as escorts and 2 as strikers.

Assuming that the escorts have a superior air-to-air combat performance, suppose the input parameters are estimated as follows:

P_r - The probability that the interceptors are directed to one of the escort elements = .8

P_{BVR} - The probability of detecting the interceptors with the on-board radar by the escort aircraft at BVR = .75

P_{BVR} - The probability of detecting the interceptors with the on-board radar by the strike aircraft at BVR = .2

P_{VIS} - The probability of visually detecting the interceptors = .9

P_{RHAW} - The probability of detecting the interceptors with the help of RHAW (Radar Warning And Homing system), by the escort aircraft = .6

P_{RHAWB} - The probability of detecting the interceptors with the help of RHAW, by the strike aircraft = .6

P_{DVT} - The probability that a strike element can dodge the interceptors after detecting them visually, and can continue with the mission, given that the escorts are tied = .8

P_{DVF} - The probability that a strike element can dodge the interceptors after detecting them visually, and can continue with the mission, given that the escorts are free = .2

P_{DR} - The probability that the strike element can dodge the interceptors after detecting them with the radar, given that the escorts are free = .7

$P_{EN}(1)$ - The probability of "1" escort aircraft being destroyed in a neutral setup against the interceptors = .18

$P_{EN}(2) = .01$

$P_{SN}(1)$ - The probability of "1" strike aircraft being destroyed in a neutral setup against the interceptors = .5

$P_{SN}(2) = .25$

$P_{ED}(1)$ - The probability of "1" escort aircraft being destroyed in a defensive setup against the interceptors = .42

$$P_{ED}(2) = .09$$

$P_{ED}(1)$ - The probability of "1" strike aircraft being destroyed in a defensive setup against the interceptors = .42

$$P_{ED}(2) = .49$$

$P_{4V2}(1)$ - The probability of "1" strike aircraft being destroyed in a 4-verses-2 setup against the interceptors = .095

$$P_{4V2}(2) = .0025$$

$P_K(1)$ - The probability of "1" aircraft being destroyed, given that the interceptor's attack was undetected = .7

$$P_K(2) = .2$$

During the ingress phase, a total of three interceptions are expected. The decision maker wants to find out the expected number of aircraft reaching the target and the expected number of aircraft destroyed during the three interceptions.

The model was run with the estimated data and the summary of the output results is shown in Table 5.1.

OPTION (ES,ST)	EXPECTED NUMBER OF STRIKE AIRCRAFT OVER THE TARGET	EXPECTED NUMBER OF AIRCRAFT DESTROYED
1. (0,8)	3.51633	1.92204
2. (2,6)	2.92446	1.54327
3. (4,4)	2.32274	1.15531
4. (6,2)	1.72263	0.728874

TABLE 5.1 THE EXPECTED OUTCOMES AFTER 3 INTERCEPTIONS

From the output results, it can be noted that with fewer escorts the expected aircraft attrition as well as the expected number of strike aircraft reaching the target is higher. Figures 5.1 and 5.2 show the graphic depiction of the expected outcomes.

Figure 5.3 is an alternate way of visualizing the output results--a plot of "strike aircraft over the target" verses "the number of aircraft that survive the interceptor threat."

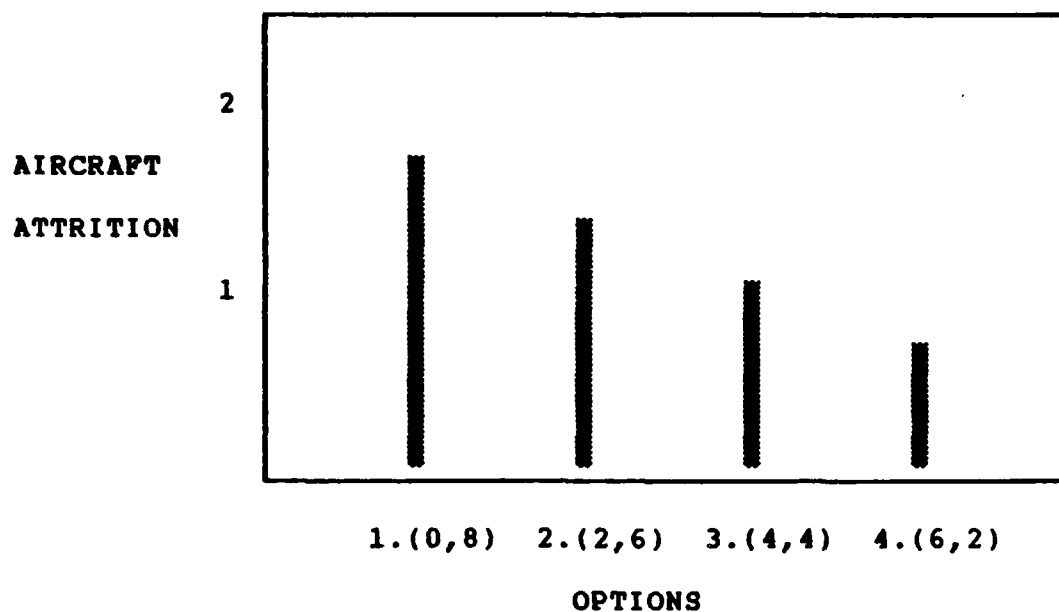


FIGURE 5.1 EXPECTED AIRCRAFT ATTRITION (WITH A FIXED TOTAL NUMBER OF AIRCRAFT)

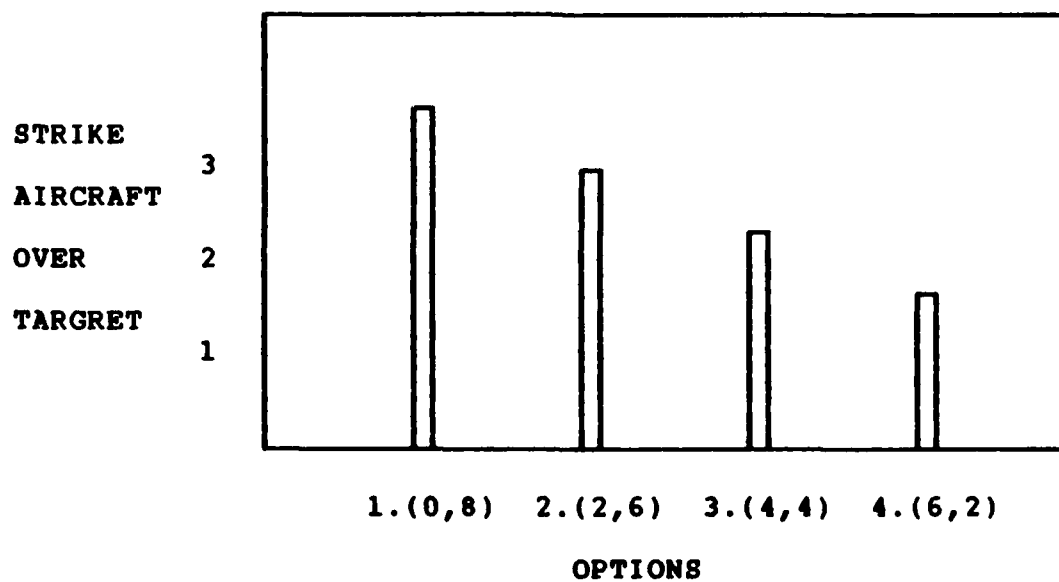
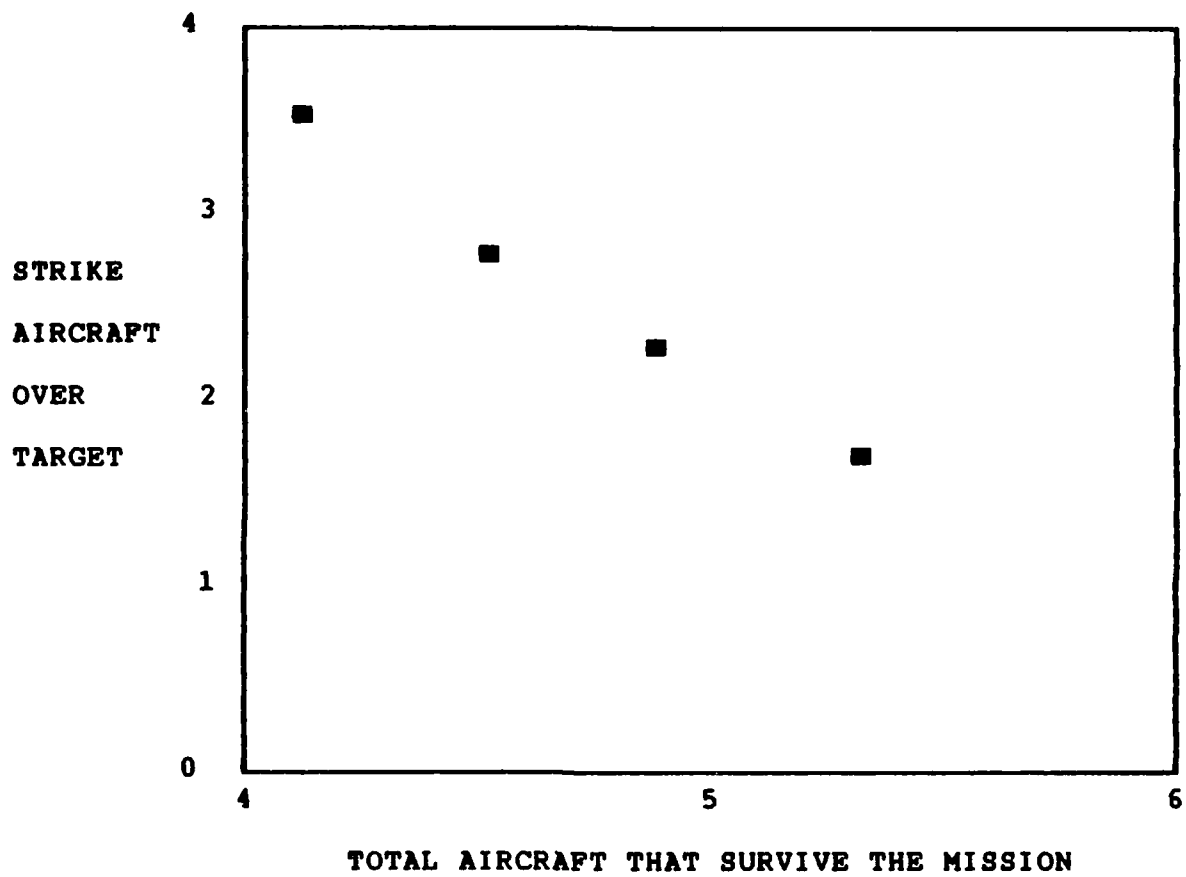


FIGURE 5.2 EXPECTED NUMBER OF STRIKE AIRCRAFT OVER THE TARGET (WITH A FIXED TOTAL NUMBER OF AIRCRAFT)



**FIGURE 5.3 AIRCRAFT OVER TARGET VERSES AIRCRAFT
 THAT SURVIVE**

EXAMPLE 5.1b

Suppose that the number of strike aircraft is fixed to "4", and the expected outcomes are to be determined with a different number of escort aircraft with the following options:

1. No escort aircraft.
2. 2 escort aircraft.
3. 4 escort aircraft.

The summary of the output results, for this case, is shown in Table 5.2. The corresponding graphic depiction of their expected outcomes is shown in figures 5.4 and 5.5.

OPTION (ES,ST)	EXPECTED NUMBER OF STRIKE AIRCRAFT OVER THE TARGET	EXPECTED NUMBER OF AIRCRAFT DESTROYED
1. (0,4)	0.422637	1.62473
2. (2,4)	1.05761	1.54341
3. (4,4)	2.32274	1.15531

**TABLE 5.2 THE EXPECTED OUTCOMES AFTER 3 INTERCEPTIONS
(WITH A FIXED NUMBER OF STRIKE AIRCRAFT)**

It can be noted that the expected aircraft attrition decreases and the expected number of strike aircraft reaching the target increases with an increase in the number of escorts. The model provides a precise estimate of the expected outcomes.

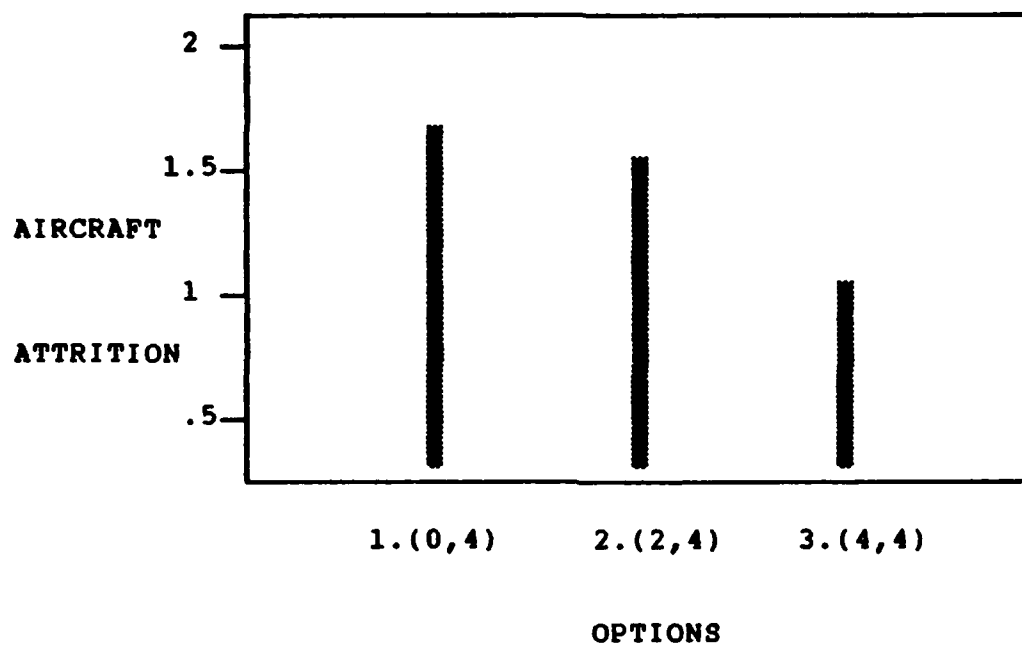


FIGURE 5.4 EXPECTED AIRCRAFT ATTRITION (WITH A FIXED NUMBER OF STRIKE AIRCRAFT)

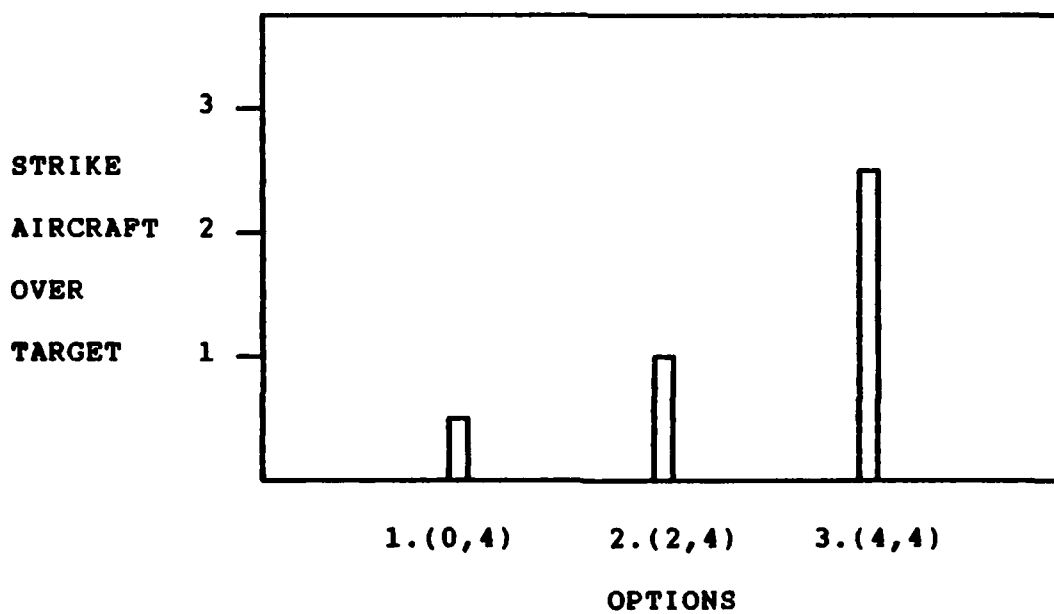


FIGURE 5.5 EXPECTED NUMBER OF STRIKE AIRCRAFT OVER THE TARGET (WITH A FIXED NUMBER OF STRIKE AIRCRAFT)

VI. DECISION MAKING

INTRODUCTION

The air combat model, presented in this paper, enables a decision maker to determine the expected outcomes of a certain number of interceptions against a tactical strike mission. The outcomes basically depend on the "mix" of the two types of aircraft--the escorts and the strikers. Since a decision maker may have multiple objectives, it would be desirable to select the most suitable combination of the two types of aircraft and strike a balance between the achievement of separate objectives. This chapter deals with the subject of decision making with the stated purpose.

THE CRITERIA FOR A DECISION

It can be assumed that an ultimate objective of a tactical strike mission would be to gain advantage over the enemy. The achievement of this aim requires destruction of the assigned target at minimum cost. Cost, which is the aircraft attrition in this case, would be an important criteria in this context because the decision maker would like to maximize his resources for the forthcoming operations. Therefore, the sub-objectives before the decision maker would be as follows:

- (a) Maximum damage to the enemy.
- (b) Minimize aircraft losses.

The two sub-objectives conflict with each other in a sense that one is achieved at the cost of the other. However, the

priority of one may vary with respect to the other depending upon the urgencies existing at that particular time. For example, in an extreme case, a decision maker may be willing to loose all his aircraft to destroy a particular target. While on the other hand, in a war of attrition, the survival of the resources may be the ultimate aim.

Since the decision involves multiple criteria, The Analytical Hierarchy Procedure (AHP) is considered suitable, in this case, for decision making.

ANALYTIC HIERARCHY PROCESS (AHP)

The AHP involves development of a hierarchy of the main objective and the sub-objectives with weights assigned to each sub-objective. The options are first evaluated in terms of the sub-objectives at the lowest level of the hierarchy and then evaluated in terms of the main objective (7).

The main objective and the two sub-objectives are already stated in this case. Suppose, W_a and W_b are the "weights" assigned by the decision maker to the two sub-objectives: "(a)" and "(b)", where

$$W_a + W_b = 1$$

Also suppose that he has "n" different options available, with the values assigned to the options, in terms of the two sub-objectives, as:

$$V^a_1, V^a_2, V^a_3, \dots, V^a_n$$

and

$$V^b_1, V^b_2, V^b_3, \dots, V^b_n \quad \text{respectively (see Fig. 5.1)}$$

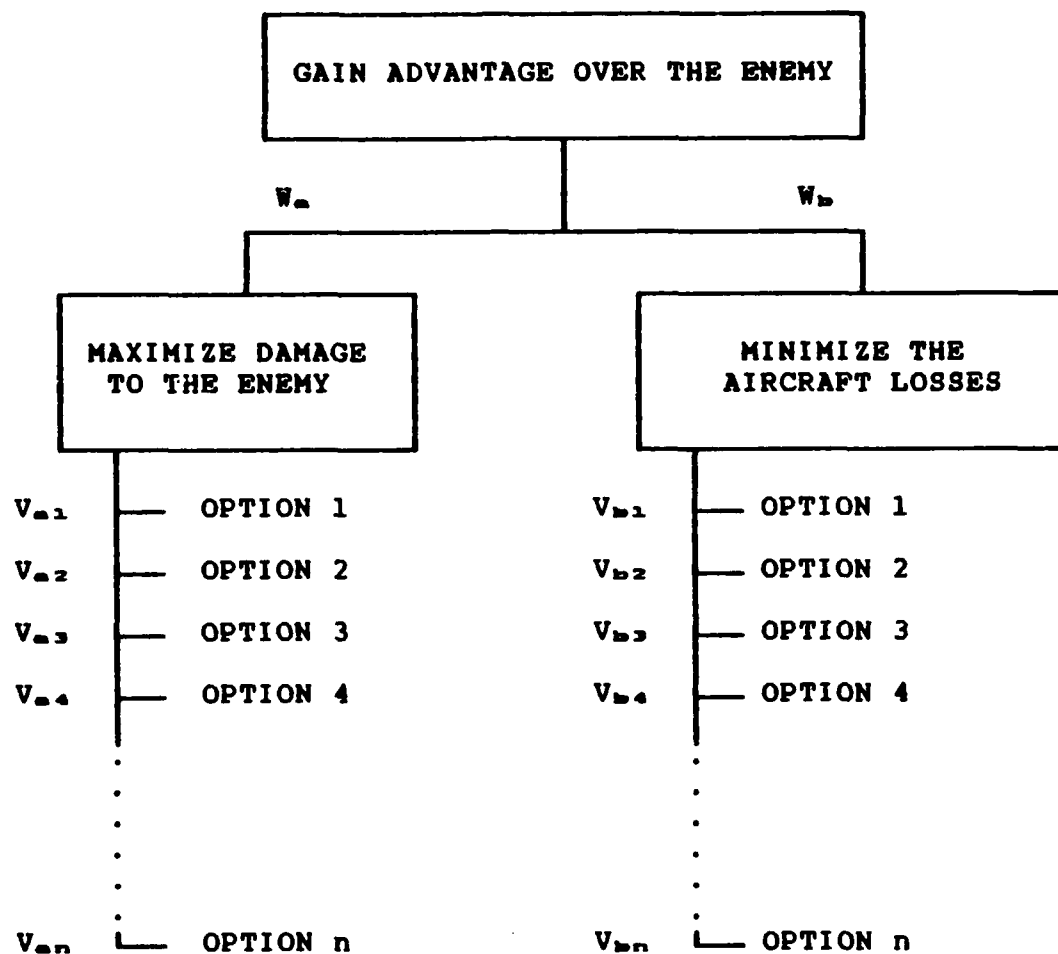


FIG. 6.1 THE ANALYTICAL HIERARCHY

where

$$V^a_1 + V^a_2 + V^a_3 + \dots + V^a_n = 1$$

and

$$V^b_1 + V^b_2 + V^b_3 + \dots + V^b_n = 1$$

The values of the options, in terms of the main objective are given by

$$(W_a.V^a_1 + W_b.V^b_1), (W_a.V^a_2 + W_b.V^b_2), \dots (W_a.V^a_n + W_b.V^b_n) \quad 6.1$$

The decision can then be based on the final values of the available options.

THE OPTIONS AND THEIR EVALUATION

The "options" available to a decision maker are the various combinations of the two types of aircraft. The model, in this case can determine the expected number of aircraft destroyed, and the expected number of aircraft on target, for each mix of the two types. Depending upon the outcome, each option of the "mix" will correspond to a certain "value" in terms of the either sub-objective. For example, the achievement of the Sub-objective "(a)" can be measured in terms of the number of strike aircraft on the target, and the achievement of the Sub-objective "(b)", in terms of the aircraft attrition.

EVALUATION OF AN OPTION IN TERMS OF THE SUB-OBJECTIVE "(a)"

The damage inflicted to the enemy is a function of the bomb-load delivered on the target, therefore, the number of strike aircraft reaching the target provides a direct measure of the damage inflicted. The relationship between the two, however, may

be linear or non-linear, depending on the nature of the target and the bomb-load delivered by each aircraft. An option can, therefore, be evaluated by using the relation between the damage expectancy and the number of strike aircraft reaching the target.

EVALUATION OF AN OPTION IN TERMS OF THE SUB-OBJECTIVE "(b)"

The model determines the expected number of friendly aircraft destroyed for a given option. A decision maker will have a "utility" function, relating the "loss experienced" and the number of aircraft destroyed. The decision makers utility function can be used to evaluate a certain option in terms of the Sub-objective (b). A possible technique for capturing the decision makers utility function is included in Appendix D.

EXAMPLE 6.1

Consider Example 5.1. Eight aircraft are available for a strike mission with following configuration options:

1. All eight aircraft as strikers.
2. 2 aircraft as escorts and 6 as strikers.
3. 4 aircraft as escorts and 4 as strikers.
4. 6 aircraft as escorts and 2 as strikers.

The expected outcomes after three interceptions are given in Table 6.1.

OPTION (ES,ST)	EXPECTED NUMBER OF STRIKE AIRCRAFT OVER THE TARGET	EXPECTED NUMBER OF AIRCRAFT DESTROYED
1. (0,8)	3.51633	1.92204
2. (2,6)	2.92446	1.54327
3. (4,4)	2.32274	1.15531
4. (6,2)	1.72263	0.728874

TABLE 6.1 EXPECTED OUTCOMES AFTER 3 INTERCEPTIONS

The damage expectancy has a linear relation with the number of aircraft reaching the target (Figure 6.1). The maximum damage expectancy is assumed to be "1", provided at least four aircraft make the target: and the minimum damage expectancy is assumed to be "0", if no aircraft makes the target. The decision maker's utility function for the aircraft destruction is shown in Figure 6.2. In this case the decision maker has "utility" equal to "1" for the best outcome and "0" for the worst outcome. The utility curve has a concave shape for a "typical risk-averse" decision maker.

The problem is to determine the most appropriate mix of the two types of aircraft in the offensive formation for the following three cases:

- (a) Enemy damage is "nine" times more important than the survival of the friendly aircraft.

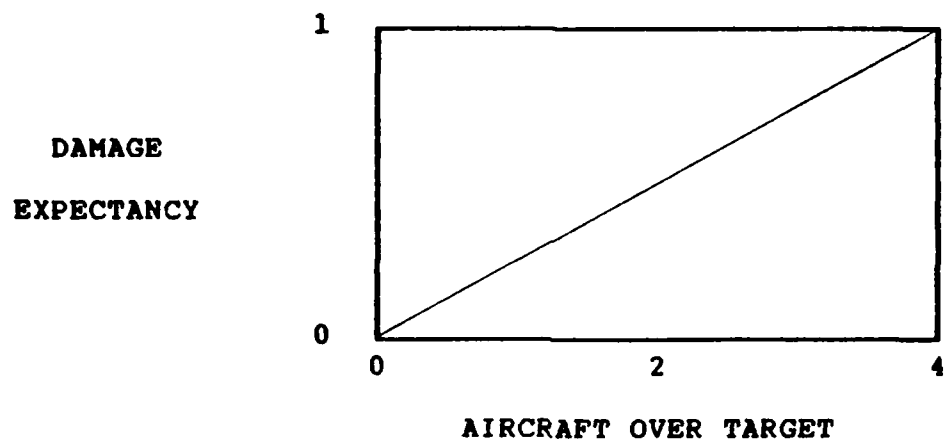
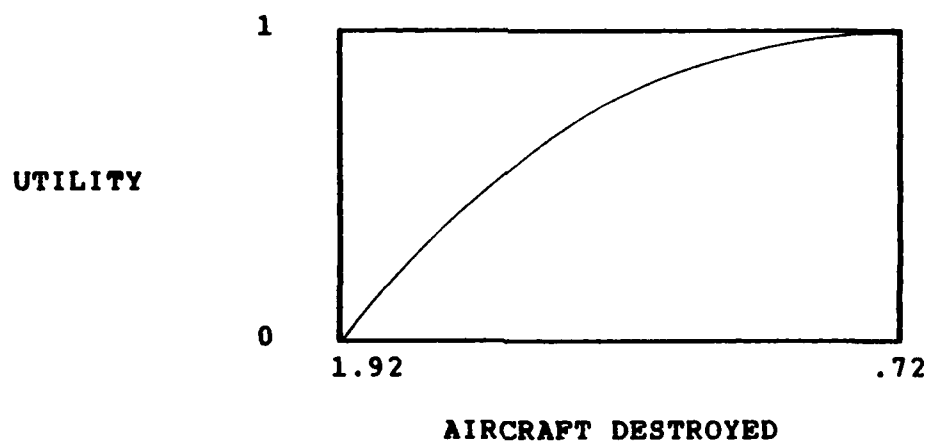


FIGURE 6.2 DAMAGE EXPECTANCY



**FIGURE 6.3 DECISION MAKER'S UTILITY CURVE
FOR AIRCRAFT LOSSES**

(b) Survival of the friendly aircraft. is "nine" times more important than the enemy damage.

(c) Enemy damage is equally important as the survival of the friendly aircraft.

The evaluation of the options in terms of damage expectancy, (from the graph in the Fig. 6.2) is shown in Table 6.2; and the decision makers "utility" for the aircraft attrition, corresponding to each option (from Fig. 6.3) is shown in Table 6.3.

OPTION	NO. OF AIRCRAFT OVER THE TGT.	DAMAGE EXPECTANCY	
		(RAW)	(NORMALIZED)
1.(0,8)	3.516	.85	.33
2.(2,6)	2.924	.72	.28
3.(4,4)	2.322	.57	.22
4.(6,2)	1.722	.41	.16

TABLE 6.2 ENEMY DAMAGE EXPECTANCY FOR THE AVAILABLE OPTIONS

OPTION	NO. OF AIRCRAFT DESTROYED	DECISION MAKER'S UTILITY	
		(RAW)	(NORMALIZED)
1.(0,8)	1.92	0	0
2.(2,6)	1.54	.67	.25
3.(4,4)	1.15	.93	.35
4.(6,2)	.72	1.0	.38

TABLE 6.3 DECISION MAKER'S UTILITY FOR THE AVAILABLE OPTIONS

The final "option scores" for the three cases, by using the Formula 6.1, are shown in Table 6.4.

OPTION	SCORE IN TERMS OF THE MAIN OBJECTIVE		
	CASE - (a)	CASE - (b)	CASE - (c)
1.(0,8)	.297 *	.033	.165
2.(2,6)	.277	.253	.265
3.(4,4)	.233	.337	.285 *
4.(6,2)	.182	.358 *	.270

TABLE 6.4 EVALUATION OF OPTIONS IN TERMS OF THE MAIN OBJECTIVE

A decision can now be based on the on the ranking of the options according to their scores in terms of the main objective. For example, option "1" is ranked best for Case-(a); option "4" is ranked best for Case-(b); and option "3" is ranked best for Case-(c).

VII. CONCLUSIONS AND RECOMMENDATIONS

The purpose of this chapter is to recap the methodology that has been described in this paper and to high-light its advantages and its limitations. Some possible areas where further work could produce significant improvements are also outlined.

SUMMARY OF THE PAPER

This paper set out to develop a methodology for determining the optimum mix of the two types, the escort and the strike aircraft, in an offensive formation. The objective was to model the likely air combat between the interceptors and the offensive formation and determine the expected outcomes after a certain number of interceptions.

The analysis is based on the assumption that in an interception against the offensive formation, the first event would be a BVR (Beyond Visual Range) missile attack by the interceptors followed by a close-range combat. The close-range air combat is broken down into a series of relatively simple events, where each event has a certain probability of occurrence. The events of the air combat form a "hierarchy". The number of aircraft destroyed and the number of aircraft available with the formation are defined as the "outcomes" of a combat. The probabilities of the outcomes are determined from the event probabilities by folding back the event hierarchy.

To account for the effect of more than one interception, the number of aircraft in the offensive formation are modelled as

a "finite state Markov Chain", with transitions occurring as a result of "engagements". The transition probabilities for the "Markov Chain" are derived from the probabilities of the outcomes of individual engagements. The number of friendly aircraft destroyed in an engagement are calculated as "expected rewards" associated with the transitions; and the expected number of aircraft available with the offensive formation are calculated from the transition probability distribution.

In order to calculate the expected outcomes of a certain number of interceptions, for a given combination of the two types of aircraft, a decision maker has to estimate the various input parameters, which are the probabilities of the basic events of the air combat and the kill probabilities of the hostile weapons.

For the purpose of decision making with the help of the output data from the model, the Analytic Hierarchy Process (AHP), based on the Multiple Criteria Decision Theory, is also outlined in the paper.

ADVANTAGES OF THE METHODOLOGY

A major advantage of this methodology is the fundamental concept on which the air combat model is based. The air combat is modelled by breaking down the complex process into simple, mutually exclusive events. The probabilities of the final outcomes are determined from the probabilities of the simple events. For further refinement, There is a possibility of breaking these events into sub-events, and thereby creating a more elaborate but an accurate hierarchy. A hierarchy with the

smallest possible detail can accurately represent any kind of air combat.

Another major advantage of this methodology is the possibility of its application in the "reverse" manner, that is, for determining the interceptor force requirement against tactical air mass raids. In that case, the interceptors can be considered on the friendly side and the offensive formations on the enemy side.

The model can also be expanded to include the remaining air defense threats: the barrier SAMs, the random area SAMs and the terminal defenses. In this way a more comprehensive picture of the expected outcomes of a strike mission can be obtained.

LIMITATIONS

The model rests on certain basic assumptions. For example, it is assumed that:

- (a) The interceptor aircraft will fly as pairs and multiple pairs will be used against a large size raid.
- (b) The aircraft from the offensive formation will participate in a combat as elements and not as individuals.
- (c) The surviving aircraft from an engagement will not be able to rejoin with the parent formation.

These assumptions are based on the present day concepts of air warfare. The model will not be valid if these assumptions are incorrect. In that case, modifications will have to be made accordingly.

The accuracy of the output results depends on the values of

the input parameters. The methodology does not recommend any procedure for the estimation of the input parameters. An inaccurate estimate can produce misleading results. Therefore, extreme care should be exercised while estimating the input parameters.

RECOMMENDATIONS

In order to improve the model, further development in the following areas is possible:

- (a) Improve the event hierarchy by breaking down the process of air combat into more fundamental events.
- (b) Expand the model by including the remaining air defense threats like area SAMs and terminal defenses.
- (c) Devise a methodology for an accurate estimation of the input parameters.

APPENDIX A: COMPUTER PROGRAM FOR THE MODEL

```

C*****
PROGRAM STRIKE
C*****
  DIMENSION P(66,66),Q(66,66),R(66,66),AC(3,66),D(66)
  COMMON/XX1/II,PF,PBVRT,PBVRB,PVIS,PRHAW,PRHAWB,
+PDVT,PDVF,PDR
  COMMON/XX2/DPK,DEN,DED,DSN,DSB,D4V2
C*****
C [P]: 'ONE STEP' TRANSITION MATRIX FOR ONE ENGAGEMENT
C [Q]: TRANSITION MATRIX AFTER "N" ENGAGEMENTS
C R(I,J): EXPECTED NUMBER OF AIRCRAFT DESTROYED CORRESPONDING
C          TO A TRANSITION (I,J)
C [AC]: ROW-1 CONTAINS THE "STATE NUMBER"
C       ROW-2 CONTAINS THE CORRESPONDING NUMBER OF ESCORT A/C
C       ROW-3 CONTAINS THE CORRESPONDING NUMBER OF STRIKE A/C
C [D]: EXPECTED NUMBER OF AIRCRAFT DESTROYED IN A CLOSE
C       COMBAT, GIVEN THAT THE INITIAL STATE WAS "I"
C*****
  ****
C ASSIGN VALUES TO ROW # 1 OF [AC]
  ****
      DO 10 I=1,66
        AC(1,I)=I
10    CONTINUE
  ****
C ASSIGN VALUES TO ROW # 2 OF [AC]
  ****
      DATA (AC(2,I),I=1,66)/0,1,2,3,4,5,6,7,8,9,10,0,1,2,3,4,5
+6,7,8,9,0,1,2,3,4,5,6,7,8,0,1,2,3,4,5,6,7,0,1,2,3,4,5
+6,0,1,2,3,4,5,0,1,2,3,4,0,1,2,3,0,1,2,0,1,0/
  ****
C ASSIGN VALUES TO ROW # 3 OF [AC]
  ****
      DATA (AC(3,I),I=1,66)/10,9,8,7,6,5,4,3,2,1,0,9,8,7,6,5,4
+3,2,1,0,8,7,6,5,4,3,2,1,0,7,6,5,4,3,2,1,0,6,5,4,3,2,1
+0,5,4,3,2,1,0,4,3,2,1,0,3,2,1,0,2,1,0,1,0,0/
C
  PRINT*, 'THE NUMBER OF ESCORT AIRCRAFT: '
  READ*, ES
  PRINT*, 'THE NUMBER OF STRIKE AIRCRAFT: '
  READ*, ST
  PRINT*, 'ESCORTS FREE? (1 FOR YES & 0 FOR NO)'
  READ*, II
  PRINT*, 'NUMBER OF INTERCEPTIONS EXPECTED'
  READ*, KK
C
C**** CUMDST: NO. OF AIRCRAFT DESTROYED TILL NOW
CUMDST=0.
CALL IDENT(Q)

```

```

C**** JJ: THE INITIAL STATE OF THE SYSTEM
      CALL STATE(AC,ES,ST,JJ)
      PRINT*, 'THE INITIAL STATE WAS',JJ
C
      OPEN (8,FILE='Y.OUT',STATUS='NEW')
      DO 20 N=1,KK
C****  "INT1": THE INPUT DATA FILE FOR INTERCEPTION-1
      IF (N.EQ.1) OPEN (7,FILE='INT1',STATUS='OLD')
      IF (N.EQ.2) OPEN (7,FILE='INT2',STATUS='OLD')
      IF (N.EQ.3) OPEN (7,FILE='INT3',STATUS='OLD')
      IF (N.EQ.4) OPEN (7,FILE='INT4',STATUS='OLD')
      IF (N.EQ.5) OPEN (7,FILE='INT5',STATUS='OLD')
      IF (N.EQ.6) OPEN (7,FILE='INT6',STATUS='OLD')
      IF (N.EQ.7) OPEN (7,FILE='INT7',STATUS='OLD')
      IF (N.EQ.8) OPEN (7,FILE='INT8',STATUS='OLD')
      READ (7,*)PF,PBVRT,PBVRB,PVIS,PRHAW,PRHAWB,
+PDVT,PDVF,PDR,PKB,BN,PK1,PK2,PEN1,PEN2,PED1,PED2,
+PSN1,PSN2,PSD1,PSD2,P4V21,P4V22
C
      IF (BN.EQ.0) GOTO 30
      CALL BVRTR(P,AC,BN,PKB,PF,II)
      CALL BVRDST(II,CUMDST,ES,ST,BN,PKB,PF)
      CALL TRANS(Q,P)
30    CALL EXPDST(PK1,PK2,PEN1,PEN2,PED1,PED2,
+PSN1,PSN2,PSD1,PSD2,P4V21,P4V22)
      CALL CCTRANS(P,R,AC)
      CALL CCDEST(P,Q,R,D,JJ,CCDST,CUMDST)
      CALL TRANS(Q,P)
      CALL ACINTACT(Q,AC,JJ,ESAV,STAV)
      CALL OUTPUT(CUMDST,ESAV,STAV,N)
      CLOSE (7)
20    CONTINUE
      END
C
      SUBROUTINE OUTPUT(CUMDST,ESAV,STAV,N)
C*****
C  THIS SUBROUTINE WRITES THE OUTPUT DATA
C*****
      WRITE(8,*)'*****'
      WRITE(8,*)'THE EXPECTED NUMBERS AFTER INTERCEPTION:',N
      WRITE(8,*)'*****'
      WRITE(8,*)'NUMBER OF AIRCRAFT DESTROYED          =',CUMDST
      WRITE(8,*)'NUMBER OF STRIKE AIRCRAFT OVER THE TGT=',STAV
      WRITE(8,*)'NUMBER OF ESCORT AIRCRAFT AVAILABLE   =',ESAV
      RETURN
      END
C

```



```

      SUBROUTINE STATE(AC,ES,ST,JJ)
C*****
C  THIS SUBROUTINE DETERMINES THE INITIAL STATE "JJ", OF THE
C  SYSTEM
C*****
      DIMENSION AC(3,66)
      S=ES+ST
      IF (S.GT.10) THEN
        PRINT*, 'INPUT DATA NOT VALID'
        STOP
      ENDIF
      I=1
15    IF (AC(2,I).EQ.ES.AND.AC(3,I).EQ.ST) THEN
      JJ=AC(1,I)
      RETURN
    ENDIF
      I=I+1
      GOTO 15
    END

C
      SUBROUTINE IDENT(Q)
C*****
C  THIS SUBROUTINE SETS [Q]=[I]
C*****
      DIMENSION Q(66,66)
      DO 40 I=1,66
        DO 45 J=1,66
          Q(I,J)=0
          IF (I.EQ.J) Q(I,J)=1
45      CONTINUE
40      CONTINUE
      RETURN
    END

C
      SUBROUTINE TRANS(A,B)
C*****
C  THIS SUBROUTINE MULTIPLIES MATRIX [A] WITH MATRIX [B]
C  AND RETURNS [A] AS THE PRODUCT
C*****
      DIMENSION A(66,66),B(66,66),C(66,66)
      DO 500 I=1,66
        DO 510 J=1,66
          C(I,J)=0
          DO 520 K=1,66
            C(I,J)=C(I,J)+A(I,K)*B(K,J)
520      CONTINUE
510      CONTINUE
500      CONTINUE

```

```

****
C  SET [A] = [C]
****
      DO 530 I=1,66
        DO 540 J=1,66
          A(I,J)=C(I,J)
540    CONTINUE
530    CONTINUE
      RETURN
      END

C
      SUBROUTINE ACINTACT(Q,AC,JJ,ESAV,STAV)
C*****
C  THIS SUBROUTINE CALCULATES THE EXPECTED NUMBER OF ESCORT
C  AND STRIKE AIRCRAFT AVAILABLE WITH THE OFFENSIVE FORMATION
C  AFTER "N" INTERCEPTIONS
C  ESAV: THE NUMBER OF ESCORT AIRCRAFT AVAILABLE
C  STAV: THE NUMBER OF STRIKE AIRCRAFT AVAILABLE
C*****
      DIMENSION Q(66,66),AC(3,66)
      ESAV=0
      STAV=0
      DO 610 I=1,66
        ESAV=ESAV+AC(2,I)*Q(JJ,I)
        STAV=STAV+AC(3,I)*Q(JJ,I)
610    CONTINUE
      RETURN
      END

C
      SUBROUTINE BVRTR(P,AC,BN,PKB,PF,II)
C*****
C  THIS SUBROUTINE GENERATES THE "ONE STEP" TRANSITION
C  MATRIX [B] (FOR THE EFFECT OF BVR MISSILES), WHERE:
C  BN = THE NUMBER OF BVR MISSILES THAT CAN BE LAUNCHED BY
C      THE INTERCEPTORS
C  PKB = THE KILL PROBABILITY OF ONE BVR MISSILE
C  II IS THE INDICATOR WHETHER THE ESCORTS ARE "FREE" OR
C  "TIED" (II=0 IF THE ESCORTS ARE TIED & II=1 OTHERWISE)
C*****
      DIMENSION P(66,66),AC(3,66)
      IF (II.EQ.0) GOTO 240
      IF (II.EQ.1) GOTO 270
240    DO 250 I=1,66
      DO 260 J=1,66
        ****
C  X: THE NUMBER OF ESCORT AIRCRAFT BY WHICH THE FORMATION
C  REDUCES
C  Y: THE NUMBER OF STRIKE AIRCRAFT BY WHICH THE FORMATION
C  REDUCES
        ****
          X=AC(2,I)-AC(2,J)
          Y=AC(3,I)-AC(3,J)

```

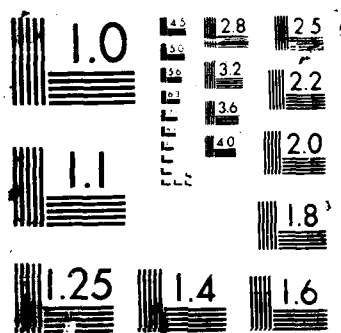
NO-AL95 873 A METHODOLOGY FOR DETERMINING THE OPTIMUM MIX OF ESCORT 2/2
AND STRIKE AIRCRA. (U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. M AVAIS
UNCLASSIFIED MAR 88 AFIT/GOR/ENS/870-1 F/G 15/6 NL

UNCLASSIFIED

MAR 88 AFIT/GOR/ENS/87D-1

F/G 15/6

NL



```

****
C   PROBABILITY OF AN INCREASE IN THE NUMBER IS "0"
****
      IF (X.LT.0.OR.Y.LT.0) THEN
        P(I,J)=0
        GOTO 260
      ENDIF

****
C   THE NUMBER OF AIRCRAFT DESTROYED CANNOT BE MORE THAN THE
C   NUMBER OF MISSILES LAUNCHED
****
      C=X+Y
      IF (C.GT.BN) THEN
        P(I,J)=0
        GOTO 260
      ENDIF
      P(I,J)=PRB(X,Y,AC(2,I),AC(3,I),PKB,BN)
260  CONTINUE
250  CONTINUE
      RETURN
C*****
C   FOR FREE ESCORTS
C*****
270  DO 275 I=1,66
      DO 280 J=1,66
        ES=AC(2,I)
        ST=AC(3,I)
        X=AC(2,I)-AC(2,J)
        Y=AC(3,I)-AC(3,J)

****
C   PROBABILITY OF AN INCREASE IN THE NUMBER IS "0"
****
      IF (X.LT.0.OR.Y.LT.0) THEN
        P(I,J)=0
        GOTO 280
      ENDIF

****
C   PROBABILITY OF DESTROYING BOTH TYPES IS "0"
****
      IF (X.GT.0.AND.Y.GT.0) THEN
        P(I,J)=0
        GOTO 280
      ENDIF

****
C   THE NUMBER OF AIRCRAFT DESTROYED CANNOT BE MORE THAN THE
C   NUMBER OF MISSILES LAUNCHED
****
      IF (X.GT.BN.OR.Y.GT.BN) THEN
        P(I,J)=0
        GOTO 280
      ENDIF
      FP=1.-PF

```

```

      IF (X.EQ.0.AND.Y.EQ.0) GOTO 271
      IF (X.GT.0.AND.Y.EQ.0) GOTO 272
      IF (X.EQ.0.AND.Y.GT.0) GOTO 273
271  P(I,J)=PRBF(0.,PKB,ES,BN,PF)+PRBF(0.,PKB,ST,BN,FP)
      GOTO 280
272  P(I,J)=PRBF(X,PKB,ES,BN,PF)
      GOTO 280
273  P(I,J)=PRBF(Y,PKB,ST,BN,FP)
      GOTO 280
280  CONTINUE
275  CONTINUE
      RETURN
      END

C
      FUNCTION PRB(A,B,EN,SN,PKB,BN)
C*****
C  THIS FUNCTION CALCULATES THE PROBABILITY OF "A" ESCORTS
C  AND "B" STRIKERS BEING DESTROYED OUT OF "EN" ESCORTS AND
C  "SN" STRIKERS, WHERE:
C  BN = NUMBER OF BVR MISSILES INSTALLED ON THE
C      INTERCEPTORS
C  PKB = KILL PROBABILITY OF ONE BVR MISSILE
C  RBN = NUMBER OF BVR MISSILES THAT CAN BE ACTUALLY
C      LAUNCHED
C  RN = TOTAL NUMBER OF AIRCRAFT IN THE OFFENSIVE FORMATION
C*****
      RBN=BN
      RN=EN+SN
      IF (RBN.GT.RN) RBN=RN
      AB=A+B
      PRB=BIN(AB,PKB,RBN)*HGP(A,EN,AB,RN)
C***** HGP(A,EN,AB,RN) = HGP(B,SN,AB,RN) *****
      RETURN
      END

C
      FUNCTION PRBF(Y,PKB,AA,RN,PF)
C*****
C  THIS FUNCTION CALCULATES THE PROBABILITY OF "Y" AIRCRAFT
C  DESTROYED WHEN "RN" BVR MISSILES ARE INSTALLED ON THE
C  INTERCEPTORS, AND THERE ARE "AA" NUMBER OF AIRCRAFT IN
C  THE TARGET FORMATION
C*****
C
C  ****
C  THE NUMBER OF AIRCRAFT DESTROYED CANNOT BE MORE THAN
C  THE MISSILES LAUNCHED
C  ****
      IF (Y.GT.AA) THEN
        PRBF=0.
        RETURN
      ENDIF
      TN=RN
      IF (RN.GT.AA) TN=AA

```

```

PRBF=PF*BIN(Y,PKB,TN)
RETURN
END

C
FUNCTION BIN(Y,P,RN)
C*****
C THIS FUNCTION CALCULATES THE PROB(Y) FOR
C A BINOMIAL DISTRIBUTION, WHERE:
C P = PROBABILITY OF SUCCESS
C RN = NUMBER OF TRAILS
C*****
IF (Y.GT.RN.OR.Y.LT.0) THEN
BIN=0
RETURN
ENDIF
EE=RN-Y
BIN=CNR(RN,Y)*(P**Y)*(1.-P)**EE
RETURN
END

C
FUNCTION HGP(Y,B,R,RN)
C*****
C THIS FUNCTION CALCULATES THE PROB(Y) FOR
C A HYPER-GEOMETRIC DISTRIBUTION, WHERE:
C RN = TOTAL NUMBER
C B = THE NUMBER OF THE TYPE OF INTEREST
C R = THE NUMBER CHOSEN OUT OF (B)
C*****
AA=B-Y
BB=RN-R
IF (R.GT.RN.OR.AA.GT.BB) THEN
HGP=0
RETURN
ENDIF
HGP=CNR(R,Y)*CNR(BB,AA)/CNR(RN,B)
RETURN
END

C
FUNCTION CNR(RN,R)
C*****
C THIS FUNCTION CALCULATES " N-CHOOSE-R "
C*****
IF (R.GT.RN) THEN
CNR=0
RETURN
ENDIF
CC=RN-R
CNR=FAC(RN)/(FAC(R)*FAC(CC))
RETURN
END

C

```

```

      FUNCTION FAC(RN)
C*****
C  CALCULATES FACTORIAL VALUE FOR A NUMBER RN
C*****
      FAC=1.
      IF (RN.EQ.0.) RETURN
      DO 700 A=1,RN
        FAC=FAC*A
700  CONTINUE
      RETURN
      END

C
      SUBROUTINE BVRDST(II,CUMDST,ES,ST,BN,PKB,PF)
C*****
C  THIS SUBROUTINE CALCULATES THE EXPECTED NUMBER OF AIRCRAFT
C  DESTROYED IN A BVR MISSILE ATTACK
C*****
      IF (II.EQ.0) GOTO 710
      IF (II.EQ.1) GOTO 711
710  TBN=BN
      SUM=ES+ST
      IF (BN.GT.SUM) TBN=SUM
      ****
C    BVDST: THE EXPECTED NUMBER OF AIRCRAFT DESTROYED DURING
C          ONE BVR MISSILE ATTACK
      ****
      BVDST=TBN*PKB
      CUMDST=CUMDST+BVDST
      RETURN
711  EBN=BN
      IF (BN.GT.ES) EBN=ES
      ****
C    EDST: THE EXPECTED NUMBER OF STRIKERS DESTROYED DURING
C          ONE BVR MISSILE ATTACK
C    SDST: THE EXPECTED NUMBER OF ESCORTS DESTROYED DURING
C          ONE BVR MISSILE ATTACK
      ****
      EDST=PF*PKB*EBN
      SBN=BN
      IF (BN.GT.ST) SBN=ST
      SDST=(1.-PF)*PKB*SBN
      CUMDST=CUMDST+EDST+SDST
      RETURN
      END
C

```



```

      SUBROUTINE CCTRANS(P,R,AC)
      DIMENSION P(66,66),R(66,66),AC(3,66)
C*****
C  THIS SUBROUTINE GENERATES THE "ONE STEP" TRANSITION
C  MATRIX [P] AND THE CORRESPONDING MATRIX [R] FOR THE EFFECT
C  OF CLOSE COMBAT
C*****
      DO 355 I=1,66
      DO 350 J=1,66
        ES=AC(2,I)
        ST=AC(3,I)
        X=AC(2,I)-AC(2,J)
        Y=AC(3,I)-AC(3,J)
        W=X+Y
        IF (X.LT.0.OR.Y.LT.0) GOTO 300
        IF (ST.EQ.0.AND.J.LT.66) GOTO 300
        IF (ST.EQ.1.AND.J.LT.66) GOTO 300
        IF (ST.EQ.1.AND.J.EQ.66) GOTO 308
        IF (ST.EQ.0.AND.J.EQ.66) GOTO 308
        IF (W.GT.6) GOTO 300
        IF (ES.EQ.0) GOTO 301
        V=MOD(ST,2.)
        IF (ES.EQ.1.AND.V.EQ.0) GOTO 309
        IF (ES.EQ.1.AND.V.EQ.1) GOTO 311
        IF (ES.GE.4.AND.ST.GE.4) GOTO 302
        IF (ES.GE.4.AND.ST.EQ.3) GOTO 303
        IF (ES.GE.4.AND.ST.EQ.2) GOTO 302
        IF (ES.EQ.3.AND.ST.GE.4) GOTO 304
        IF (ES.EQ.3.AND.ST.EQ.3) GOTO 305
        IF (ES.EQ.3.AND.ST.EQ.2) GOTO 304
        IF (ES.EQ.2.AND.ST.GE.4) GOTO 306
        IF (ES.EQ.2.AND.ST.EQ.3) GOTO 307
        IF (ES.EQ.2.AND.ST.EQ.2) GOTO 306
300    P(I,J)=0
        R(I,J)=0
        GOTO 350
301    P(I,J)=PROBU(Y,ST)
        R(I,J)=DESTU(Y,ST)
        GOTO 350
302    P(I,J)=PT(X,Y)
        R(I,J)=DST(X,Y)
        GOTO 350
303    P(I,J)=PROB(X,Y,1)
        R(I,J)=DEST(X,Y,1)
        GOTO 350
304    P(I,J)=PROB(X,Y,2)
        R(I,J)=DEST(X,Y,2)
        GOTO 350
305    P(I,J)=PROB(X,Y,3)
        R(I,J)=DEST(X,Y,3)
        GOTO 350

```

```

306  P(I,J)=PROB(X,Y,4)
      R(I,J)=DEST(X,Y,4)
      GOTO 350
307  P(I,J)=PROB(X,Y,5)
      R(I,J)=DEST(X,Y,5)
      GOTO 350
308  P(I,J)=1.
      R(I,J)=0
      GOTO 350
309  IF (X.EQ.0.AND.Y.EQ.0) GOTO 310
      IF (X.EQ.1) GOTO 315
      P(I,J)=0
      R(I,J)=0
      GOTO 350
315  IF (Y.EQ.1) GOTO 313
      IF (Y.EQ.3) GOTO 314
      IF (Y.EQ.4.AND.ST.EQ.4) GOTO 310
      IF (Y.EQ.2.AND.ST.EQ.2) GOTO 310
      P(I,J)=0
      R(I,J)=0
      GOTO 350
310  P(I,J)=PROBU(Y,ST)
      R(I,J)=DESTU(Y,ST)
      GOTO 350
314  IF (ST.NE.4) GOTO 313
      P(I,J)=0
      R(I,J)=0
      GOTO 350
311  IF (X.EQ.0.AND.Y.EQ.0) GOTO 312
      IF (X.EQ.1.AND.Y.EQ.1) GOTO 313
      IF (X.EQ.1.AND.Y.EQ.3) GOTO 313
      P(I,J)=0
      R(I,J)=0
      GOTO 350
312  P(I,J)=PROBU(0.,ST)
      R(I,J)=DESTU(0.,ST)
      GOTO 350
313  ST1=ST+1.
      P(I,J)=PROBU(W,ST1)
      R(I,J)=DESTU(W,ST1)
350  CONTINUE
355  CONTINUE
      RETURN
      END

```

C

```

      FUNCTION PROB(X,Y,J)
C*****
C  THIS FUNCTION CALCULATES THE PROBABILITY OF THE OFFENSIVE
C  FORMATION REDUCING BY "X" ESCORTS AND "Y" STRIKERS FOR A
C  CONDITION "J" SPECIFIED BY THE SUBROUTINE-CCTrans,
C  (ESCORTS AVAILABLE)
C*****
      IF (J.EQ.1) GOTO 400
      IF (J.EQ.2) GOTO 410
      IF (J.EQ.3) GOTO 420
      IF (J.EQ.4) GOTO 430
      IF (J.EQ.5) GOTO 440
400   IF (Y.EQ.0) GOTO 401
      IF (Y.EQ.3) GOTO 402
      PROB=0
      RETURN
401   PROB=PT(X,0.)
      RETURN
402   PROB=PT(X,2.)
      RETURN
410   IF (X.EQ.3.AND.Y.EQ.0) GOTO 441
      IF (X.EQ.3.AND.Y.EQ.2) GOTO 442
      IF (X.EQ.0) GOTO 450
      PROB=0
      RETURN
420   IF (X.EQ.3.AND.Y.EQ.0) GOTO 441
      IF (X.EQ.3.AND.Y.EQ.3) GOTO 442
      IF (X.EQ.0) GOTO 450
      PROB=0
      RETURN
430   IF (X.EQ.2.AND.Y.EQ.0) GOTO 441
      IF (X.EQ.2.AND.Y.EQ.2) GOTO 442
      IF (X.EQ.0) GOTO 450
      PROB=0
      RETURN
440   IF (X.EQ.2.AND.Y.EQ.0) GOTO 441
      IF (X.EQ.2.AND.Y.EQ.3) GOTO 442
      IF (X.EQ.0) GOTO 450
      PROB=0
      RETURN
441   PROB=PT(2.,0.)+PT(4.,0.)
      RETURN
442   PROB=PT(2.,2.)
      RETURN
450   IF (Y.EQ.0) GOTO 451
      IF (Y.EQ.2.OR.Y.EQ.2) GOTO 452
      PROB=0
      RETURN
451   PROB=PT(0.,0.)
      RETURN
452   PROB=PT(0.,2.)+PT(0.,4.)
      RETURN
      ENL

```

```

      FUNCTION PROBU(Y,ST)
C*****
C  THIS FUNCTION CALCULATES THE PROBABILITY OF THE OFFENSIVE
C  FORMATION REDUCING BY "X" ESCORTS AND "Y" STRIKERS FOR A
C  CONDITION "J" SPECIFIED BY THE SUBROUTINE-CCTrans,
C  (ESCORTS NOT AVAILABLE)
C*****
      IF (ST.GE.4) GOTO 190
      IF (ST.EQ.3.AND.Y.EQ.3) GOTO 191
      IF (ST.EQ.3.AND.Y.EQ.0) GOTO 190
      IF (ST.EQ.2.AND.Y.EQ.2) GOTO 191
      IF (ST.EQ.2.AND.Y.EQ.0) GOTO 190
      PROBU=0
      RETURN
190  PROBU=PTU(Y)
      RETURN
191  PROBU=PTU(2.)+PTU(4.)
      RETURN
      END

C
      FUNCTION PT(X,Y)
C*****
C  THIS FUNCTION IS USED BY FUNCTION-PROB TO CALCULATE THE
C  PROBABILITY OF THE OFFENSIVE FORMATION REDUCING BY "X"
C  ESCORTS AND "Y" STRIKERS
C*****
      COMMON/XX1/II,PF,PBvre,PBvrs,Pvis,PRHawe,PRHaws,
      +PDVT,PDVF,PDR
      IF (II.EQ.0) GOTO 110
      IF (II.EQ.1) GOTO 120
110  IF (X.EQ.0) THEN
      PT=0
      RETURN
      ENDIF
      IF (X.EQ.4.AND.Y.EQ.0) GOTO 111
      IF (X.EQ.2.AND.Y.EQ.0) GOTO 112
      IF (X.EQ.2.AND.Y.EQ.2) GOTO 113
      PT=0
      RETURN
120  IF (X.EQ.4.AND.Y.EQ.0) GOTO 111
      IF (X.EQ.2.AND.Y.EQ.0) GOTO 121
      IF (X.EQ.2.AND.Y.EQ.2) GOTO 122
      IF (X.EQ.0.AND.Y.EQ.2) GOTO 123
      IF (X.EQ.0.AND.Y.EQ.4) GOTO 124
      IF (X.EQ.0.AND.Y.EQ.0) GOTO 125
      PT=0
      RETURN
CCC
111  PT=(1.-PRHawe)*(1.-Pvis)*(1-PBvre)*PF
      RETURN
112  PTT=(1.-PRHawe)*(1.-Pvis)*(1-PBvre)*PF
      PT=PF-PTT+PBvrs*(1.-PF)+PDVT*Pvis*(1.-PBvrs)*(1.-PF)
      RETURN

```

```

113     PT =(1.-PF)*(1.-PBVRS)*((1.-PVIS) + PVIS*(1-PDVT))
        RETURN
CCC
121     PT=PF-(1.-PRHAWE)*(1.-PVIS)*(1-PBVRE)*PF
        RETURN
122     PT=0
        RETURN
123     PX=(1.-PF)*(1.-PBVRS)*(PRHAWS*(1.-PVIS)+PVIS*(1.-PDVF))

        PY = PBVRS*(1.-PDR)*(1.-PF)
        PT=PX+PY
        RETURN
124     PT=(1.-PRHAWS)*(1.-PVIS)*(1.-PBVRS)*(1.-PF)
        RETURN
125     PT=(1.-PF)*(PDR*PBVRS+PDVF*PVIS*(1.-PBVRS))
        RETURN
END

C
      FUNCTION PTU(Y)
C*****
C  THIS FUNCTION IS USED BY FUNCTION-PROB TO CALCULATE THE
C  PROBABILITY OF THE OFFENSIVE FORMATION REDUCING BY
C  "Y" STRIKERS (ESCORTS NOT AVAILABLE)
C*****
      COMMON/XX1/II,PF,PBVRE,PBVRS,PVIS,PRHAWE,PRHAWS,
+PDVT,PDVF,PDR
      IF (Y.EQ.0) GOTO 141
      IF (Y.EQ.2) GOTO 142
      IF (Y.EQ.4) GOTO 143
      PTU=0
      RETURN
141     PTU=(PDR*PBVRS+PDVF*PVIS*(1.-PBVRS))
      RETURN
142     PX=(1.-PBVRS)*(PRHAWS*(1.-PVIS)+PVIS*(1.-PDVF))
      PY = PBVRS*(1.-PDR)
      PTU=PX+PY
      RETURN
143     PTU=(1.-PRHAWS)*(1.-PVIS)*(1.-PBVRS)
      RETURN
END

```

```

      FUNCTION DEST(X,Y,J)
C*****
C  THIS FUNCTION CALCULATES THE EXPECTED NUMBER OF AIRCRAFT
C  IF THE OFFENSIVE FORMATION REDUCES BY "X" ESCORTS AND "Y"
C  STRIKERS FOR A CONDITION "J" SPECIFIED BY THE
C  SUBROUTINE-CCTrans (ESCORTS AVAILABLE)
C*****
      IF (J.EQ.1) GOTO 800
      IF (J.EQ.2) GOTO 810
      IF (J.EQ.3) GOTO 820
      IF (J.EQ.4) GOTO 830
      IF (J.EQ.5) GOTO 840

800   IF (Y.EQ.0) GOTO 801
      IF (Y.EQ.3) GOTO 802
      DEST=0
      RETURN
801   DEST=DST(X,0.)
      RETURN
802   DEST=DST(X,2.)
      RETURN
CCC
810   IF (X.EQ.3.AND.Y.EQ.0) GOTO 841
      IF (X.EQ.3.AND.Y.EQ.2) GOTO 842
      IF (X.EQ.0) GOTO 850
      DEST=0
      RETURN
820   IF (X.EQ.3.AND.Y.EQ.0) GOTO 841
      IF (X.EQ.3.AND.Y.EQ.3) GOTO 842
      IF (X.EQ.0) GOTO 850
      DEST=0
      RETURN
830   IF (X.EQ.2.AND.Y.EQ.0) GOTO 841
      IF (X.EQ.2.AND.Y.EQ.2) GOTO 842
      IF (X.EQ.0) GOTO 850
      DEST=0
      RETURN
840   IF (X.EQ.2.AND.Y.EQ.0) GOTO 841
      IF (X.EQ.2.AND.Y.EQ.3) GOTO 842
      IF (X.EQ.0) GOTO 850
      DEST=0
      RETURN
841   DEST=DST(2.,0.)+DST(4.,0.)
      RETURN
842   DEST=DST(2.,2.)
      RETURN
850   IF (Y.EQ.2.OR.Y.EQ.2) THEN
      DEST=DST(0.,2.)+DST(0.,4.)
      ENDIF
      DEST=0
      RETURN
      END

```

```

      FUNCTION DESTU(Y,ST)
C*****
C  THIS FUNCTION CALCULATES THE EXPECTED NUMBER OF AIRCRAFT
C  IF THE OFFENSIVE FORMATION REDUCES BY "Y" STRIKERS FOR A
C  CONDITION "J" SPECIFIED BY THE
C  SUBROUTINE-CCTRANS (ESCORTS AVAILABLE)
C*****
      IF (ST.GE.4) GOTO 890
      IF (ST.EQ.3.AND.Y.EQ.3) GOTO 891
      IF (ST.EQ.2.AND.Y.EQ.2) GOTO 891
      DESTU=0
      RETURN
890  DESTU=DSTU(Y)
      RETURN
891  DESTU=DSTU(2.)+DSTU(4.)
      RETURN
      END

C
      SUBROUTINE EXPDST(PK1,PK2,PEN1,PEN2,PED1,PED2,
+PSN1,PSN2,PSD1,PSD2,P4V21,P4V22)
C*****
C  THIS SUBROUTINE CALCULATES THE EXPECTED NUMBER OF FRIENDLY
C  AIRCRAFT DESTROYED, GIVEN THE PROBABILITY DISTRIBUTION OF
C  THE NUMBER OF AIRCRAFT DESTROYED
C*****
      COMMON/XX2/DPK,DEN,DED,DSN,DSD,D4V2
      DPK=2*PK2+PK1
      DEN=2*PEN2+PEN1
      DED=2*PED2+PED1
      DSN=2*PSN2+PSN1
      DSD=2*PSD2+PSD1
      D4V2=2*P4V22+P4V21
      RETURN
      END

C
      FUNCTION DST(X,Y)
C*****
C  THIS FUNCTION IS USED BY FUNCTION-DEST TO CALCULATE THE
C  EXPECTED NUMBER OF AIRCRAFT DESTROYED IF THE OFFENSIVE
C  FORMATION REDUCING BY "X" ESCORTS AND "Y" STRIKERS
C*****
      COMMON/XX1/II,PF,PBVRT,PBVR,PVIS,PRHAW,PRHAW,
+PDVT,PDVF,PDR
      COMMON/XX2/DPK,DEN,DED,DSN,DSD,D4V2
      IF (II.EQ.0) GOTO 910
      IF (II.EQ.1) GOTO 920
910  IF (X.EQ.0) THEN
      DST=0
      RETURN
      ENDIF
      IF (X.EQ.4.AND.Y.EQ.0) GOTO 911
      IF (X.EQ.2.AND.Y.EQ.0) GOTO 912
      IF (X.EQ.2.AND.Y.EQ.2) GOTO 913

```

```

DST=0
RETURN
920 IF (X.EQ.4.AND.Y.EQ.0) GOTO 911
    IF (X.EQ.2.AND.Y.EQ.0) GOTO 921
    IF (X.EQ.0.AND.Y.EQ.2) GOTO 922
    IF (X.EQ.C.AND.Y.EQ.4) GOTO 923
DST=0
RETURN
911 DST=DPK*(1.-PRHAW)* (1.-PVIS)*(1.-PBVRE)*PF
    RETURN
C
912 DD1=DEN*PF*PBVRE
    DD2=DED*PF*(1.-PBVRE)*(PVIS+(1.-PVIS)*PRHAW)
    DD3=DED*(1.-PF)*PBVRS
    DST=DD1+DD2+DD3
    RETURN
C
913 DD4=(1.-PF)*(1.-PBVRS)
    DD5=DD4*(1.-PVIS)*(PRHAW*DSD+(1.-PRHAW)*DPK)
    DD6=DD4*PVIS*(1.-PDVT)*D4V2
    DD7=DD4*PVIS*(PDVT)*DEN
    DST=DD5+DD6+DD7
    RETURN
C
921 DDA1=DEN*PF*PBVRE
    DDA2=DED*PF*(1.-PBVRE)*(PVIS+(1.-PVIS)*PRHAW)
    DST=DDA1+DDA2
    RETURN
C
922 DDA3=DSN*(1.-PF)*PBVRS*(1.-PDR)
    DDA4=DSD*(1.-PF)*(1.-PBVRS)*PVIS*(1.-PDVF)
    DDA5=DSD*(1.-PF)*(1.-PBVRS)*(1.-PVIS)*PRHAW
    DST=DDA3+DDA4+DDA5
    RETURN
C
923 DST=DPK*(1.-PRHAW)*(1.-PVIS)*(1.-PBVRS)*(1.-PF)
    RETURN
    END
C
    FUNCTION DSTU(Y)
C*****
C THIS FUNCTION IS USED BY FUNCTION-DEST TO CALCULATE THE
C EXPECTED NUMBER OF AIRCRAFT DESTROYED IF THE OFFENSIVE
C FORMATION REDUCES BY "X" ESCORTS AND "Y" STRIKERS
C (ESCORTS NOT AVAILABLE)
C*****
    COMMON/XX1/II,PF,FBVRE,PBVR,PVIS,PRHAW,PRHAW,
    +PDVT,PDVF,PDR
    COMMON/XX2/DPK,DEN,DED,DSN,DSD,D4V2
    IF (Y.EQ.2) GOTO 930
    IF (Y.EQ.4) GOTO 931
    DSTU=0
    RETURN

```



```

930   DDD1=(1.-PBVRS)*(1.-PVIS)*PRHAWSDSD
      DDD2=(1.-PBVRS)*PVIS*(1.-PDVF)*DSD
      DDD3=PBVRS*(1.-PDR)*DSN
      DSTU=DDD1+DDD2+DDD3
      RETURN
CCC
931   DSTU=DPK*(1.-PRHAWSDSD)*(1.-PVIS)*(1.-PBVRS)
      RETURN
      END
C
      SUBROUTINE CCDEST(P,Q,R,D,JJ,CCDST,CUMDST)
C*****
C   THIS SUBROUTINE CALCULATES THE EXPECTED NUMBER OF AIRCRAFT
C   DESTROYED IN A CLOSE AIR COMBAT
C*****
      DIMENSION P(66,66),Q(66,66),R(66,66),D(66)
      WRITE(8,*)' '
      WRITE(8,*)' '
      WRITE(8,*)' '
      DO 950 I=1,66
        D(I)=0
        DO 960 J=1,66
          *****
          C   D(I): EXPECTED NUMBER OF AIRCRAFT DESTROYED, GIVEN THAT
          C   THE INITIAL STATE (AT THE START OF THE MISSION)
          C   WAS "I"
          *****
          D(I)=D(I)+P(I,J)*R(I,J)
960    CONTINUE
          WRITE (8,*)D(I)
950    CONTINUE
          WRITE (8,*)' '
          CCDST=0
          DO 970 I=1,66
            *****
            C   CCDST: EXPECTED NUMBER OF AIRCRAFT DESTROYED DURING ONE
            C   CLOSE COMBAT
            *****
            CCDST=CCDST+Q(JJ,I)*D(I)
970    CONTINUE
            CUMDST=CUMDST+CCDST
            C   WRITE(8,*)'CCDST=',CCDST
            C   WRITE(8,*)'CUMDST=',CCDST
            RETURN
            END

```

APPENDIX B: A SPECIMEN OF THE INPUT FILE FOR THE
COMPUTER MODEL

INPUT FILE NAME: INT3

CONTENTS:

.8,.75,.4,.9,.6,.6,.8,.2,.7,0,0,.42,.49,.18,.001,.42,.09,.5,.25,
.42,.49,.095,.0025

VARIABLES READ:

PF, PBVRE, PBVRS, PVIS, PRHAW, PRHWS, PDVT, PDVF, PDR, PKB, BN,
PK1, PK2, PEN1, PEN2, PED1, PED2, PSN1, PSN2, PSD1, PSN2, P4V21
and P4V22.

(For the definition of the variables, see Page 44).

APPENDIX C: A SPECIMEN OUTPUT REPORT OF
THE COMPUTER MODEL

```
*****
THE EXPECTED OUTCOMES AFTER INTERCEPTION: 1
*****
NUMBER OF AIRCRAFT DESTROYED      = 0.247769
NUMBER OF STRIKE AIRCRAFT OVER THE TGT= 3.91040
NUMBER OF ESCORT AIRCRAFT AVAILABLE = 1.98400

*****
THE EXPECTED OUTCOMES AFTER INTERCEPTION: 2
*****
NUMBER OF AIRCRAFT DESTROYED      = 0.511256
NUMBER OF STRIKE AIRCRAFT OVER THE TGT= 3.80955
NUMBER OF ESCORT AIRCRAFT AVAILABLE = 0.000000

*****
THE EXPECTED OUTCOMES AFTER INTERCEPTION: 3
*****
NUMBER OF AIRCRAFT DESTROYED      = 1.15531
NUMBER OF STRIKE AIRCRAFT OVER THE TGT= 2.32274
NUMBER OF ESCORT AIRCRAFT AVAILABLE = 0.000000

*****
THE EXPECTED OUTCOMES AFTER INTERCEPTION: 4
*****
NUMBER OF AIRCRAFT DESTROYED      = 1.77285
NUMBER OF STRIKE AIRCRAFT OVER THE TGT= 1.01205
NUMBER OF ESCORT AIRCRAFT AVAILABLE = 0.000000

*****
THE EXPECTED OUTCOMES AFTER INTERCEPTION: 5
*****
NUMBER OF AIRCRAFT DESTROYED      = 2.10898
NUMBER OF STRIKE AIRCRAFT OVER THE TGT= 0.273129
NUMBER OF ESCORT AIRCRAFT AVAILABLE = 0.000000
```

APPEDIX D: FIVE- POINT ASSESMENT PROCEDURE FOR CONSTRUCTING
THE UTILITY FUNCTION FOR A DECISION MAKER

The procedure for constructing the "utility function" for a decision maker is as follows:

- STEP-1. Identify the best and the worst outcomes.
- STEP-2. Set the utility of the best outcome " $u(\text{best})$ " = 1
and the utility of the worst outcome " $u(\text{worst})$ " = 0
- STEP-3. Find " x " such that $u(x) = .5$
- STEP-4 Find " x " such that $u(x) = .25$
- STEP-5 Find " x " such that $u(x) = .75$

" $u(x)$ " can now be plotted as a function of " x " by joining the known points (4:188-196).

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END

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